Homework to be done by November 18th

Section 7.6: Problems 13, 19, 29.
Section 7.7: Problems 21, 25, 27, 37, 41, 43, 49, 55, 60, 67.
Section 7.8: Problems 5, 9, 17, 29, 37, 43, 48.
Section 7.9: Problems 14, 19, 20, 21, 37.

Problems to be handed in on November 18th

Problem 1. Consider the differential equation

$$(*) \qquad y'' + \lambda y = 0.$$

1.1 Check that $y = ax$ satisfies $(*)$ when $\lambda = 0$. Here $a \in \mathbb{R}$.

1.2 Check that $y = a\cos(\sqrt{\lambda}\,x) + b\sin(\sqrt{\lambda}\,x)$ satisfies $(*)$ when $\lambda > 0$. Here $a,\ b \in \mathbb{R}$.

1.3 Check that $y = a\cosh(\sqrt{-\lambda}\,x) + b\sinh(\sqrt{-\lambda}\,x)$ satisfies $(*)$ when $\lambda < 0$. Here $a,\ b \in \mathbb{R}$.

Problem 2. Let

$$f(x) = \tan^{-1}\left(\frac{a+x}{1-ax}\right), \quad x \neq \frac{1}{a}.$$

2.1 Show that

$$f'(x) = \frac{1}{1+x^2}, \quad x \neq \frac{1}{a}.$$

2.2 Show that there is no constant C such that

$$f(x) = \tan^{-1} x + C \quad \text{for all} \quad x \neq \frac{1}{a}.$$

2.3 Find constants C_1 and C_2 such that

$$f(x) = \tan^{-1} x + C_1 \quad \text{for} \quad x > \frac{1}{a},$$

and

$$f(x) = \tan^{-1} x + C_2 \quad \text{for} \quad x < \frac{1}{a}.$$