Homework to be done by November 4th

Section 5.5: Problems 11, 16, 29, 48, 50.
Section 5.6: Problems 19, 27, 36, 38, 47.
Section 5.7: Problems 1-6, 14, 15, 21, 22, 34.
Section 5.8: Problems 17, 18, 25.
Section 6.1: Problems 35, 37.
Section 6.2: Problems 5, 13, 21, 27, 37, 43.
Section 6.3: Problems 5, 31, 34.

Another problem to think about:

1. Compute $\int_{-1}^{1} x^{-2}\,dx$ by mindlessly applying the fundamental theorem of calculus.
2. Sketch the graph of $y = x^{-2}$. By interpreting the integral as the area under the graph, show that your answer to question 1 is ridiculous.
3. What went wrong?

Problems to be handed in on November 4th

Problem 1. Let

$$F(x) = \int_{\cos(\mathrm{x}^2)}^{\sqrt{x}} \frac{dt}{\sqrt{t^4+1}}.$$

Compute that $F'(x)$.

Problem 2. Assume that $f, g : [a, b] \to \mathbb{R}$ are continuous and that $f', g' : [a, b] \to \mathbb{R}$ are also continuous. Show that

$$\int_a^b f'(x)g(x)\,dx = f(b)g(b) - f(a)g(a) - \int_a^b f(x)g'(x)\,dx$$

Problem 3.

3.1 Suppose that f is continuous function on the real line $\mathbb{R}$. Prove that for every $x \in \mathbb{R}$

$$\lim_{h\to 0} \frac{1}{h} \int_x^{x+h} f(t)\,dt = f(x).$$

(Hint: Express the average value in terms of the antiderivative of f. No handwaving please.)

3.2 Suppose that f and g are continuous functions on the real line that have the same average value on every interval. Show that $f = g$. (Use 3.1.)

Problem 4. A wedge is formed by slicing through a circular cylindrical block of radius r with a plane that passes through the diameter of the base of the cylinder and makes an angle of $\frac{\pi}{6}$ with the base. Find the volume of the wedge.