Homework to be done by October 7th

Section 2.6: Problems 2, 3, 7, 11, 13, 15.
Section 3.1: Problems 9, 11, 19, 24, 27, 30, 34, 36.
Section 3.2: Problems 27, 30, 54, 57.
Section 3.3: Problems 43, 46, 49, 61, 62.

Another problem to think about: Suppose that f is differentiable at $x = a$. Which of the following formulas for $f'(a)$ are correct?

$$f'(a) = \lim_{h \to 3} \frac{f(a+h-3) - f(a)}{h-3}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a) - f(a-h)}{h}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+2h) - f(a)}{h}$$

If the formula is incorrect, what is the limit on the right?

Problems to be handed in on October 7th

1. Suppose that f is a continuous function on [0,1] that takes values in [0,1]. (That is $0 \leq f(x) \leq 1$ for $0 \leq x \leq 1$.) Show that there us at least one point c in [0,1] such that $f(c) = c$. (c is called a *fixed point* of f.) Hint: Consider $g(x) = f(x) - x$.

2. Let

$$f(x) = \begin{cases} x^3 - 2 & \text{if } x \leq 2 \\ Ax + \frac{B}{x} & \text{if } x > 2, \end{cases}$$

where A and B are constants.

2.1 What relationship must A and B satisfy for f to be continuous at $x = 2$?
2.2 Precisely one choice of A and B will make f differentiable at $x = 2$. Find it.

3. Let n be a positive integer.
3.1 Prove that if $0 \leq a < b$ then $a^n < b^n$. Hint: Use mathematical induction.
3.2 Prove that every nonnegative real number x has a unique nonnegative n^{th} root $x^{1/n}$. Hint: The existence of $x^{1/n}$ can be seen by applying the intermediate value theorem to the function $f(t) = t^n$ for $t \geq 0$. The uniqueness follows from 2.1.