Homework to be done by December 6th

Section 8.5: Problems 3, 7, 8, 10, 26, 27, 33, 34, 37.
Section 8.6: Problems 9, 10, 11, 13, 16, 19, 21, 23, 25, 27, 44.
Section 8.7: Problems 4(a), 7(a),(b), 11.
Section 8.8: Problems 2, 6, 13, 14, 17, 19, 21, 25, 35.
Section 8.9: Problems 2, 3, 7, 11, 13, 14, 15, 19, 21, 27.

Problems to be handed in on December 6th

1. Evaluate the following integral:

$$\int \frac{4\arctan x}{(x+1)^3}\,dx.$$

2. Mr. X invests money in a savings account that pays $100r\%$ interest per year, compounded continuously (i.e. if the interest rate is 8% then $r = 0.08$). At time $t = 0$ (t being measured in years), she makes an initial deposit of 500 dollars, and thereafter invests continuously at a rate of $500(1 + 0.05t)$ dollars per year. His rate of investment increases with time, as his salary grows larger. Of course the continuous compounding and continuous depositing are not quite true in life, but they are good approximations in the long run.

2.1 Set up and solve the differential equation for $M(t)$, the amount of money in the account at time t.

2.2 How much money will he save after 30 years if the interest rate is 5%? if the interest rate is 10%?

3. Consider a chemical reaction in which a molecule of compound P combines with a molecule of compound Q to form a molecule of a new compound X. Let $x(t)$ be the amount of X at time t, and let p and q be the initial amounts of P and Q. The amounts of P and Q at time t are $p - x(t)$ and $q - x(t)$. Suppose that the reaction proceeds at a rate that is jointly proportional to the amounts of P and Q:

$$\frac{dx}{dt} = a(p - x(t))(q - x(t)),$$

where a is a positive constant.

3.1 Solve the equation for $x(t)$, assuming that $x(0) = 0$. You will need to consider the case $p = q$ separately, as the form of the solution is different in this case.

3.2 Show that $x(t)$ approaches the constant $c = \min\{p, q\}$ as $t \to \infty$. Why is this what you would expect?