E-problems

1. Stein & Shakarchi: Chapter 3: problem 6 (page 154)

2. Stein & Shakarchi: Chapter 6: problem 1 (page 319)

3. A **Hamel Basis** for a vector space X is a collection $\mathcal{H}$ of vectors in X, such that any $x \in X$ can be written as a unique *finite* linear combination of elements in $\mathcal{H}$. Prove that an infinite dimensional Banach space cannot have a countable Hamel basis. [Hint: Show that otherwise the Banach space would be of first category.]

   For an excellent discussion on basis including the construction of a Hamel Basis see here.