HOMEWORK 8

TI problems due Wednesday March 3 at 10pm PT

Problems from Folland: Chapter 6, section 2: problems 20 (not the part when $p = \infty$), 21 and 22.

Problem 1: (Prelim 2006) Let μ be a Borel measure in $\mathbb{R}^d$. Suppose there exists a constant $C > 1$ such that such for each $x \in \mathbb{R}^d$ and $r > 0$

$$C^{-1} r^d \leq \mu(B(x, r)) \leq C r^d.$$

- Prove that μ is absolutely continuous with respect to the Lebesgue measure m in $\mathbb{R}^d$.
- Let f denote the Radon-Nikodym derivative of μ with respect to m, i.e. $d\,\mu = f\, d\, m$. Show that for m-a.e. $x \in \mathbb{R}^d$

$$\frac{C^{-1}}{\omega_d} \leq f(x) \leq \frac{C}{\omega_d},$$

where ω_d denotes the Lebesgue measure of the unit ball in $\mathbb{R}^d$.