R problem set 6

Due Friday March 5

Problem 1: Let (X, d) be a metric space. Show that:

1. If $x, y \in X$ and $x \neq y$ there are disjoint open sets U and V such that $x \in U$ and $y \in V$. Every metric space is Hausdorff or T_2

2. If $A, B \subset X$ are disjoint closed sets there are disjoint open sets U and V such that $A \subset U$ and $B \subset V$. Every metric space is normal or T_4.

Problem 2: Let (X, d) and (Y, ρ) be a metric spaces. Let $f, g : X \to Y$ be continuous maps. Show that:

1. $\{x \in X : f(x) = g(x)\}$ is a closed subset of X.

2. If $f = g$ on a dense subset of X, then $f = g$ on all of X.