R problem 4

Due Friday February 5 - Monday February 8

Problem 1: Let $\mathcal{H}$ be a separable Hilbert space. Let $T \in \mathcal{L}(\mathcal{H}, \mathcal{H})$. Prove that there exists a unique $T^* \in \mathcal{L}(\mathcal{H}, \mathcal{H})$ satisfying:

1. $\langle Tf, g\rangle = \langle f, T^*g\rangle$ for all f,$g \in \mathcal{H}$.
2. $\|T\| = \|T^*\|$.
3. $(T^*)^* = T$.

Problem 2: Let $\mathcal{H}$ be a separable Hilbert space. Let $T \in \mathcal{L}(\mathcal{H}, \mathcal{H})$. We say T is symmetric if $T^* = T$. Show that if T is symmetric then

$$\|T\| = \sup\{|\langle Tf, f\rangle| : \|f\| = 1\}.$$