R problem 3

Due Friday February 5

Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be separable Hilbert spaces, $U \in \mathcal{L}(\mathcal{H}_1, \mathcal{H}_2)$ is **unitary** if it is a bijection and

$$\|Uf\|_{\mathcal{H}_2} = \|f\|_{\mathcal{H}_1} \text{ for all } f \in \mathcal{H}_1.$$

Problem 1: Show that any two infinite-dimensional separable Hilbert spaces are unitarily equivalent. (See the Polarization identity in Folland Chapter 5 problem 55 a.)

Problem 2: Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be separable Hilbert spaces. Show that for $T \in \mathcal{L}(\mathcal{H}_1, \mathcal{H}_2)$

$$\|T\| = \sup\{|\langle Tf, g\rangle| : f \in \mathcal{H}_1,\ g \in \mathcal{H}_2,\ \|f\|_1 \leq 1,\ \|g\|_2 \leq 1\}.$$