HOMEWORK 2

**TI** problems due Wednesday January 20 at 10pm PT

**Problems from Folland:** Chapter 3: 17. Chapter 5: 11 and 12 (ignore 12e).

**Problem 1:** For $f \in L^1_{loc}(\mathbb{R}^n)$, and $0 < s < n$ define

$$\Lambda_s = \{x \in \mathbb{R}^n : \liminf_{r \to 0} \frac{1}{r^s} \int_{B(x,r)} |f|(y)\, dy > 0.\}$$

Prove that $m(\Lambda_s) = 0$, for all $s \in (0, n)$.

**Problem 2:** Let $(X, d)$ be a metric space, $(Y, \rho)$ a complete metric space, and $\{f_n\}_n$ a sequence of functions from $X$ to $Y$ such that

$$\sup_{x \in X} \rho(f_n(x), f_m(x)) \to 0 \text{ as } m,\ n \to \infty.$$

There is a unique function $f : X \to Y$ such that

$$\sup_{x \in X} \rho(f_n(x), f(x)) \to 0 \text{ as } n \to \infty.$$

If each $f_n$ is continuous, so is $f$.