WHAT DISTINGUISHES $\mathbb{Q}$ FROM $\mathbb{R}$?

Complete after reading Construction of the real numbers

COMPLETENESS AXIOM: Every non-empty set $S \subset \mathbb{R}$ which is bounded above has a least upper bound.

1. Define *upper bound* for a non-empty set $S \subset \mathbb{R}$.
2. Define *least upper bound* for a non-empty set $S \subset \mathbb{R}$.

Prove the following consequences of the Completeness axiom:

- Axiom of Archimedes: Given any real number x there is an integer n such that $x < n$.
- Between any two real numbers there is always a rational number, i.e. given $x, y \in \mathbb{R}$ with $x < y$ there is $r \in \mathbb{Q}$ such that $x < r < y$.
- Let $n \in \mathbb{N}$ and $n \geq 1$, for $x > 0$ there exists a unique $y > 0$ such that $y^n = x$.
- Let $r \in \mathbb{R}^+$ and $r > 0$, for $x > 0$ there exists a unique $y > 0$ such that $y^r = x$.

QUESTION: Is there another way to express the completeness of $\mathbb{R}$? One that may apply to $\mathbb{R}^n$ or other metric spaces?