R problems

Due Monday November 9

Exercises from Folland:

Chapter 2: Section 3: problems 25 and 27.

Problem 1: Let $E \subset \mathbb{R}^n$ be a Lebesgue measurable set with $m(E) < \infty$. Show that for every $\epsilon > 0$, there exists a finite union $F = \bigcup_{j=1}^{N} Q_j$ of cubes such that

$$m(E \Delta F) < \epsilon.$$

Problem 2: A function $\Phi : \mathbb{R}^n \to \mathbb{R}$ is a step function if it can be written as a finite linear combination of characteristic functions of rectangles. Show that the following families of functions are dense in $L^1(\mu)$.

- The step functions.
- The continuous functions with compact support.