HOMEWORK 6

TI problems due Wednesday November 11 at 10pm PT

Exercises from Folland:

Chapter 2: Section 3, problems 20 and 26 .

Chapter 2: Section 4, problems 36 and 44.

Problem 1: (Prelim 2014) Let $L^1(\mathbb{R}^d)$ be the set of Lebesgue integrable functions in $\mathbb{R}^d$, let m be the Lebesgue measure in $\mathbb{R}^d$ and $\|f\|_1 = \int_{\mathbb{R}^d} |f|\, dm$. Let $\{f_n\}_{n\geq 1}$ be a sequence in $L^1(\mathbb{R}^d)$. Suppose that there exists $f \in L^1(\mathbb{R}^d)$ such that $f_n \to f$ m-a.e. and $\|f_n\|_1 \to \|f\|_1$.

1. Show that for all Lebesgue sets $E \subset \mathbb{R}^d$
$$\int_E |f_n|\, dm \to \int_E |f|\, dm \text{ as } n \to \infty.$$
2. Prove that
$$\|f_n - f\|_1 \to 0 \text{ as } n \to \infty.$$
3. Show that for every continuous function with compact support $\varphi \in C_c(\mathbb{R}^d)$
$$\lim_{n\to\infty} \int f_n\, \varphi\, dm = \int f\, \varphi\, dm$$

Remark: Problem 1 is worth 10 points.