R problems

Due Wednesday October 14

Problem 1: Show that given a collection $\{\mathcal{M}_\alpha\}_\alpha$ of σ-algebras then $\bigcap_\alpha \mathcal{M}_\alpha$ is a σ-algebra.

Problem 2: Given $\mathcal{C} \subset 2^X$ a non-empty collection of subsets on X show that the σ-algebra generated by $\mathcal{C}$ is the intersection of all σ-algebras containing $\mathcal{C}$.

Problem 3: Given a metric space (X, ρ) an outer (exterior) measure μ_* on X is said a **metric outer measure** if

$$\mu_*(A \cup B) = \mu_*(A) + \mu_*(B)$$

whenever $d(A, B) > 0$. Recall $d(A, B) = \inf\{\rho(a, b) : a \in A, b \in B\}$.

Show that if μ_* is a metric outer measure, then all closed sets are μ_*-measurable. (Hint: see class notes from October 12).