Analysis - Fall 2020

Some of the questions under consideration in 1870 (the decade this course starts)

① Suppose $\{f_n\}$ is a sequence of continuous functions in $[0,1]$. Assume that $\lim_{n \to \infty} f_n(x) = f(x)$ exists for every $x \in [0,1]$.

What can we say about f?

In 1872 Weierstrass built a "nowhere" differentiable function

$$f(x) = \sum_{n=0}^{\infty} a^n \cos(b^n x) \quad \text{with} \quad ab > 1 + \frac{3}{2}\pi$$

note f is the limit of $f_k(x) = \sum_{n=0}^{k} a^n \cos(b^n x)$ smooth.

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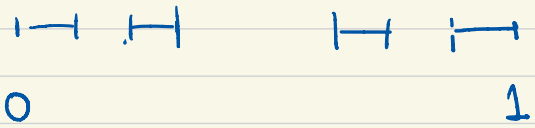
What can we say about f?

② The Riemann - Stieltjes integral is only defined for functions with countably many discontinuities.

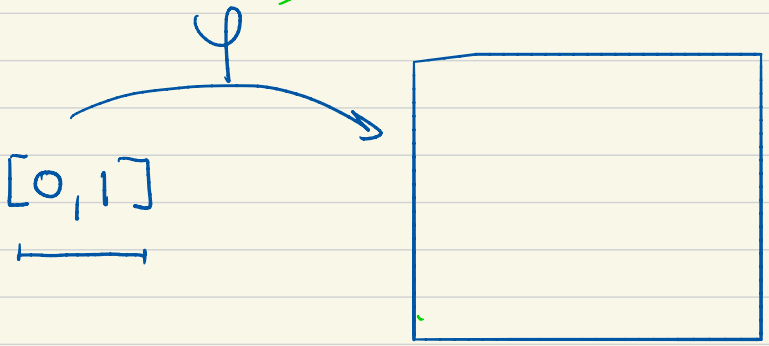
③ The notion of length in $\mathbb{R}$ well understood, what are the analogous notions in $\mathbb{R}^2$ (area) and in $\mathbb{R}^3$ (volume)

④ Strange sets (Cantor's ternary set 1883) and functions (Peano space filling curve 1890) are built.

Cantor ternary set



continuous



Peano filling curve

wrap around to fill the whole square

1898 Borel measurable sets

1902 Lebesgue theory of measure & integration

1905 Vitali : construction of non-measurable sets

Question : Is there another way to express the completeness of $\mathbb{R}$?

> Cauchy criterion : Every Cauchy sequence in $\mathbb{R}$ converges.