GEOMETRIC MEASURE THEORY

HOMEWORK

Problem 1: Show that if E is a (Λ, r_0)-perimeter minimizer in A open set in $\mathbb{R}^n$, then $\mathbb{R}^n \setminus E$ is also a (Λ, r_0)-perimeter minimizer in A.

Problem 2: Assume that $\{E_k\}_k$ is a sequence of (Λ_k, r_k)-perimeter minimizers in an open set $A \subset \mathbb{R}^n$ with $\Lambda_k r_k \leq 1$, with

$$\Lambda = \limsup_{k\to\infty} \lambda_k < \infty, \qquad r_0 = \liminf_{k\to\infty} r_k > 0.$$

Show that if A_0 is an open set compactly contained in A with $P(A_0) < \infty$ such that

$$A_0 \cap E_k \to E,$$

where E is a set of finite perimeter, then E is a a (Λ, r_0)-perimeter minimizer in A_0. Moreover

$$\mu_{A_0 \cap E_k} \rightharpoonup \mu_E \qquad \text{and} \qquad \|\partial(A_0 \cap E_k)\| \rightharpoonup \|\partial E\|.$$

Problem 3: Let $f : \mathbb{R}^n \to \mathbb{R}$ be such that for $x \in \mathbb{R}$ and $r \in (0, 1)$

$$⨍_{B(x,r)} |f(y) - f_{x,r}|\, dy \leq \kappa r^\alpha \qquad \text{where} \qquad f_{x,r} = ⨍_{B(x,r)} f(y)\, dy,$$

where $\kappa > 0$ and $\alpha \in (0, 1)$. Show that there exists a constant $C_0 = C(n, \kappa) > 0$ such that for all $x,\ y \in \mathbb{R}^n$, $|x - y| \leq 1$

$$|f(x) - f(y)| \leq C_0 |x - y|^\alpha.$$