Geometry of measures

In the 1920's Besicovitch studied linearly measurable sets in the plane, that is sets with locally finite "length". The basic question he addressed was whether the infinitesimal properties of the "length" of a set E in the plane yield geometric information on E itself. This simple question marks the beginning of the study of the geometry of measures and the associated field known as Geometric Measure Theory (GMT).

In this series of lectures we will present some of the main results in the area concerning the regularity of the support of a measure in terms of the behavior of its density or in terms of its tangent structure. We will discuss applications to PDEs, free boundary regularity problem and harmonic analysis. The aim is that the GMT component of the mini-course will be self contained.

References:

P. Mattila. Geometry of sets and measures in Euclidean spaces, Cambridge Stud. Adv. Math. 44, Cambridge Univ. Press, Cambridge, 1995.

D. Preiss, Geometry of measures in $\mathbb{R}^n$: distribution, rectifiability, and densities, Ann. of Math. 125 (1987), 537-643.

D. Preiss, X. Tolsa and T. Toro, On the smoothness of Hölder doubling measures, Calculus of Variations and PDE's 35 (2009), 339-363.

List of topics:

- Rectifiability of sets and measures
- Tangent measures
- Cones of measures
- Preiss rectifiability theorem