Math 526

Homework due 05/21/2014

Reading from Folland 2nd edition: Chapter 8: §1, §2 and §3.

Exercises from Folland: Chapter 8: 4, 8.

Problem 1 (Prelim 2007): Let E and F be complex Banach spaces, and let $T : E :\to F$ be a continuous surjective linear map. Let $T^* : F^* \to E^*$ denote the adjoint map on dual spaces, defined by $T^*f(x) = f(Tx)$ for $x \in E$ and $f \in F^*$. Prove that there is a constant $C > 0$ such that $\|T^*f\| \geq C\|f\|$ for all $f \in F^*$.

Problem 2 (Prelim 2007): Let M be a compact metric space with distance function d and let Ω denote the set of all continuous functions $f : [0, 1] \to M$ such that

$$d(f(s), f(t)) \leq |s - t| \quad \text{for all} \quad s, t \in [0, 1]$$

Define a metric ρ on Ω by the formula

$$\rho(f, g) = \sup_{0 \leq t \leq 1} d(f(t), g(t))$$

(you do not need to prove that ρ is a metric). Show that (Ω, ρ) is sequentially compact.

Problem 3 (Prelim 2005): In this problem you can use the fact that simple functions are dense in $L^2([0, 1])$.

1. Show that $C([0, 1])$ is dense in $L^2([0, 1])$ (in the L^2 norm).
2. Show that there is an orthonormal basis $\{f_n\}_{n=0}^{\infty}$ for $L^2([0, 1])$ such that f_n is a polynomial of degree n for each n.