Math 526

Homework due 05/14/2014

Reading from Stein & Shakarchi: Chapter 4: section 6.

Exercises from Stein & Shakarchi: Chapter 4: 7, 25, 28, 33.

Problem 1 (Prelim 2008): Suppose that $\mathcal{H}$ is a separable real Hilbert space with orthonormal basis $\{e_k\}_{k\geq 1}$ and with inner product denoted by $\langle \cdot, \cdot \rangle$. Let $\{y_k\}_{k\geq 1} \subset \mathcal{H}$. Prove that the following statements are equivalent

1. $\lim_{k\to\infty} \langle x, y_k \rangle = 0$ for every $x \in \mathcal{H}$.
2. $\sup_k \|y_k\| < \infty$ and $\lim_{k\to\infty} \langle e_n, y_k \rangle = 0$ for every $n \geq 1$.