Math 525

Prelim problem 2010

Let μ be a finite Borel signed measure in [0,1]. Assume that

$$\int e^{-nx}\, d\mu = 0 \text{ for all } n \in \mathbb{N}.$$

Prove that $\mu = 0$.