Math 525

Homework due 03/10/2010

Reading from Folland: Chapter 4: sections 5 and 7.

Problems from Folland: Chapter 4 : 56, 63, 64 and 68.

Problem 1: Let

$$\mathcal{H} = \left\{ u : \mathbb{R}^n \to \mathbb{R} : u \text{ continuous and } \forall x \in \mathbb{R}^n, \forall r > 0 : \ u(x) = \frac{1}{m(B(x,r))} \int_{B(x,r)} u(y)\, dy \right\}$$

Show that if $\{u_n\}_{n=1}^{\infty} \subset \mathcal{H}$ is uniformly bounded on compact subsets of $\mathbb{R}^n$, then there exists a subsequence that converges uniformly on compact subsets of $\mathbb{R}^n$ to a function $u \in \mathcal{H}$.