# Math 525

## Homework due 02/17/2010

**Reading from Folland:** Chapter 4: sections 1, 2 and 4.

**Problem 1:** Let $(X, \mathcal{M}, \rho)$ be a $\sigma$-finite (positive) measure. Prove that

$$L^2(\rho) = \{f : X \to \mathbb{R} : \int |f|^2 \, d\rho < \infty\}$$

with the inner product defined by

$$(f, g) = \int f g \, d\rho$$

satisfies the definition of Hilbert space (Chapter 4, section 2 in Stein & Shakarchi) except possibly for the separability.

**Note:** The Riesz Representation Theorem holds for such spaces.

**Problems from Folland:** Chapter 4 : 2, 3, 4 and 5.