Definitions

- **Definition:** A topological space X is locally compact if for every $x \in X$ there exist an open set $U \subset X$ such that $x \in U$ and $\overline{U}$ is compact.

- **LCH:** locally compact Hausdorff

Properties

- **Proposition:** If X is LCH and $U \subset X$ is open with $x \in U$ there exists N open such that $x \in N$, $\overline{N}$ compact and $\overline{N} \subset U$.

- **Proposition:** If X is LCH, $K \subset U \subset X$, U open and K compact there exists a precompact open set V such that

$$K \subset V \subset \overline{V} \subset U.$$

Separation by continuous functions

Urysohn Lemma, LCH version: Let X be a LCH space. Let $K \subset U \subset X$, with U open and K compact. There exists $f \in C(X, [0,1])$ such that $f = 1$ on K and $f = 0$ outside a compact subset of U.

Tietze Extension Theorem, LCH version: Let X be a LCH space. Let $K \subset X$ be compact. If $f \in C(K, [0,1])$, there exists $F \in C(X, [0,1])$ such that $F_{|K} = f$. Moreover F may be taken to vanish outside a compact set.

Continuous functions on LCH spaces

- **Proposition:** If X is a LCH space, $C_0(X) = \overline{C_c(X)}$ in the uniform metric.
- **Lemma:** If X is a LCH space and $E \subset X$, then E is closed if and only if $E \cap K$ is closed for every compact set $K \subset X$.
- **Proposition:** If X is a LCH space, $C(X)$ is a closed subset of $\mathbb{R}^X$ in the topology of uniform convergence on compact sets.