Math 525

Homework due 01/08/03

Reading: Chapter 5 of Royden. Sections 1, 2, 4 and 5, Chapter 3 from Folland.

Problem 1 Let μ be a finite Borel measure on $\mathbb{R}^n$. Let

$$S_\infty = \{x \in \mathbb{R}^n : \limsup_{\rho \to 0} \frac{\mu(B(x,\rho))}{\rho} = \infty\}.$$

Prove that for each $\epsilon > 0$ there exists a covering of S_∞ by balls $B(x_j, \rho_j)$ such that

$$\sum_j \rho_j < \epsilon.$$

Remark: This shows that S_∞ has 1-dimensional Hausdorff measure 0.