Math 524

Homework due 10/02/02

Reading: Chapter 1, Royden. Sections 1, 2, 3 and 4 in Chapter 2 in Royden. Sections 1, 2, 3, 4 and 5 in Chapter 0 in Folland.

Problem 1. Fix $b > 1$.

1.1 If $m, n, p, q \in \mathbb{Z}$, $n > 0, q > 0$, and $r = m/n = p/q$, prove that

$$(b^m)^{1/n} = (b^p)^{1/q}.$$

Hence it makes sense to define $b^r = (b^m)^{1/n}$

1.2 Prove that

$$b^{r+s} = b^r b^s$$

if $r, s \in \mathbb{Q}$.

1.3 If $x \in \mathbb{R}$ define

$$B(x) = \{b^t : t \in \mathbb{Q} \text{ and } t \leq x\}.$$

Prove that

$$b^r = \sup B(r),$$

when $r \in \mathbb{Q}$. Hence it makes sense to define

$$b^x = \sup B(x),$$

for $x \in \mathbb{R}$.

1.4 Prove that

$$b^{x+y} = b^x b^y,$$

for all $x, y \in \mathbb{R}$.

Problems from Royden Chapter 2, Section 4: problems 11 and 12.