Math 524

Homework due 03/07/01

Problem 1. Let $1 \leq p < \infty$, and $\alpha \in (0,1)$. Let $u \in L^p(\mathbb{R}^n)$ satisfy

$$\int_{B(x,r)} |u - u_{x,r}|^p \, dy \leq r^{n+p\alpha},$$

where $B(x,r) \subset \mathbb{R}^n$ denotes the ball of center x and radius $r > 0$, and $u_{x,r}$ denotes the average of u on $B(x,r)$.

1.1 Let $\rho \in (0,1)$, and for $j \geq 1$ let $r_j = \rho^j$. Show that (after possibly redefining u in a set of measure 0) for $x \in \mathbb{R}^n$

$$|u(x) - u_{x,r_j}| \leq C r_j^{\alpha},$$

where $C > 0$ is a constant that depends on n and ρ but not on j.

1.2. Show that for $r \in (0,1)$, and $x \in \mathbb{R}^n$

$$|u(x) - u_{x,r}| \leq C r^{\alpha},$$

where $C > 0$ is a constant that depends on n and ρ but not on r.

1.3 Show that $u \in C^{0,\alpha}_{loc}$. More precisely prove that there is a constant $C > 0$ depending only on n such that for $x, y \in \mathbb{R}^n$, with $|x - y| \leq 1$

$$|u(x) - u(y)| \leq C|x - y|^{\alpha}.$$

Problems from Folland

Chapter 4, Section 1: problems 3, 5, 6, 7, 10.