Math 524

Homework due 02/21/01

Problem 1. Let $(X, \mathcal{M}, \mu)$ be a measure space. Let $f \in L^r \cap L^\infty$ for some $r \in [1, \infty)$ show that

$$\lim_{p \to \infty} \|f\|_p = \|f\|_\infty.$$

Problem 2.(Prelim) Suppose that $\sup_n \|f_n\|_p < \infty$ and $f_n \to f$ a.e.

2.1 If $1 < p < \infty$ then $f_n \rightharpoonup f$ in L^p, i.e. $\forall g \in L^q$

$$\int f_n \, g \to \int f g.$$

(Hint: See problem 20 Chapter 6, section 2 in Folland.)

2.2 Show that the result above is false in general for $p = 1$.

$(*)$ **Problem 3. Boundedness of weakly convergent sequences** Let $1 < p < \infty$. Assume that $f_n \rightharpoonup f$ in L^p. Then

3.1 $\{f_n\}_{n=1}^\infty$ is bounded in L^p.

3.2 $\|f\|_p \leq \liminf_{n \to \infty} \|f_n\|_p$.

Problems from Folland

Chapter 6, Section 2 : problems 17, 22