Math 524

Homework due 01/17/01

∗**Problem.** Let $U \subset \mathbb{R}^n$ be open, and let $\delta > 0$. Show that there exists a countable disjoint collection $\mathcal{G}$ of closed balls in U such that $\operatorname{diam} \mathrm{B} \leq \delta$ for all $B \in \mathcal{G}$ and

$$m\left(U - \bigcup_{B \in \mathcal{G}} B\right) = 0.$$

∗**Problem.** A positive measure μ on $\mathbb{R}^n$ is said to be a doubling measure if there exist $C_0 > 1$ such that

$$\mu(B(x, 2r)) \leq C_0 \mu(B(x, r)), \quad \forall x \in \mathbb{R}^n \text{ and } \forall r > 0.$$

For $f \in L^1_{loc}(\mu)$ define the Hardy-Littlewood maximal function Mf by

$$Mf(x) = \sup_{r>0} \frac{1}{\mu(B(x,r))} \int_{B(x,r)} |f(y)| \, d\mu(y).$$

Let μ be a doubling measure on $\mathbb{R}^n$. Show that there exists a constant $C > 0$ such that for all $f \in L^1(\mu)$ and all $\alpha > 0$,

$$\mu\left(\{x \in \mathbb{R}^n : Mf(x) > \alpha\}\right) \leq \frac{C}{\alpha} \int_{\mathbb{R}^n} |f(y)| \, d\mu(y).$$

Problems from Folland

Chapter 3, Section 1 : problems 2, 3.
Chapter 3, Section 2 : problems 8, 11, 13.