Math 524

Homework due 01/10/01

Problem 1. (Prelim) Let $f : (0, \infty) \to \mathbb{R}$ be in $L^1(m)$. For $x > 0$ define

$$g(x) = \int_{\sqrt{x/2}}^{\infty} \frac{f(t)}{t^2}\, dt.$$

Show that $g \in L^1(m)$, where m denotes the Lebesque measure, and that

$$\int_0^{\infty} g(x)\, dx = 2 \int_0^{\infty} f(x)\, dx.$$

Problem 2. (Prelim) Let $f : [0, 1] \to \mathbb{R}^+$ be Lebesgue measurable. Let

$$A = \{g : [0, 1] \to \mathbb{R} : g \in L^1(m),\ |g| \leq f a.e. \text{ on } [0, 1]\}.$$

2.1 Show that A is a closed subset of L^1.

2.2 Show that if A is compact in $L^1(m)$ then f must be in $L^1(m)$.

Problem 3. (Prelim) Show that for every continuous real valued function f defined on [-1,1]

$$\lim_{n\to\infty} \frac{\int_{-1}^{1} f(x)x^{2n}\, dx}{\int_{-1}^{1} x^{2n}\, dx}$$

exists. Give the value of the limit in terms of f.

Problems from Folland

Chapter 3, Section 5: problem 41, 42.