Participation Homework 1: due Friday Oct 7 in class

(Note: you are also responsible for Webassign homework due Thursday)

1. Read through Example 4 in Section 1.2 of the textbook, which is on pages 19-21, and make sure you understand each step of the process, both what they are doing (which elementary operation they are using) and why they are doing it (how does it get them closer to being in echelon form?).

2. For each of the following two matrices, use Gaussian elimination to put them into echelon form. At each step, explain clearly what you are doing (which elementary operation you are using) and why you are doing it. (Putting a pivot in the right place, zeroing out terms below the pivot, etc.) Tip: Give yourself plenty of space.

3. For each of the matrices, consider the corresponding linear system. What are all possible solutions? Anything else interesting about these systems that you'd like to add?

$$\begin{bmatrix} 1 & -1 & -3 & -1 & -1 \\ -2 & 2 & 6 & 2 & -1 \\ -3 & -3 & 10 & 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & -9 & -4 & 0 \\ -3 & -11 & 9 & -1 & 0 \\ 1 & 4 & -2 & 1 & 0 \end{bmatrix}$$

Your answers should be handwritten neatly (or typed, but this isn't necessary) and turned in in class on Friday.

Problem 1 Solutions

$$\begin{bmatrix} 1 & -1 & -3 & -1 & -1 \\ -2 & 2 & 6 & 2 & -1 \\ -3 & -3 & 10 & 0 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -3 & -1 & -1 \\ 0 & 0 & 0 & 0 & -3 \\ 0 & -6 & 4 & -3 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -3 & -1 & -1 \\ 0 & -6 & 4 & -3 & 2 \\ 0 & 0 & 0 & 0 & -3 \end{bmatrix}$$

The system corresponding to this augmented matrix is inconsistent. (Note: I didn't explain all of the steps - if you have any questions about anything please let me know.)

Problem 2 Solutions

$$\begin{bmatrix} 2 & 6 & -9 & -4 & 0 \\ -3 & -11 & 9 & -1 & 0 \\ 1 & 4 & -2 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & -2 & 1 & 0 \\ -3 & -11 & 9 & -1 & 0 \\ 2 & 6 & -9 & -4 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 4 & -2 & 1 & 0 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & -2 & -5 & -6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & -2 & 1 & 0 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & -2 & 0 \end{bmatrix}$$

This is in echelon form, and we can find the solutions to the corresponding system by using back substitution. We get

$$\begin{aligned} x_4 &= s \\ x_3 &= 2s \\ x_2 &= -8s \\ x_1 &= 35s \end{aligned}$$

Notice that this system is homogeneous, and so it had better be the case that $(0, 0, ..., 0)$ is one of our solutions. Indeed it is for $s = 0$.
Also notice that even though we got infinitely many solutions, we can still check that these solutions actually satisfy the original system.
For example, we plug our general solution in to the first equation to get

$$2(35s) + 6(-8s) - 9(2s) - 4(s) = 70s - 48s - 18s - 4s = 0.$$

Notice that this holds for any s. (If you also check the next two equations, you also get zero, as desired.) Checking your answers is always a good idea.