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Modeling Roundabout Traffic Flow as a Dynamic Fluid System

Abstract

With increasing usage of roundabouts as traffic control mechanisms, it is important to develop a criteria for the design of efficient roundabouts. We designate vehicle throughput as the primary measure of a roundabouts efficiency and delay experienced by vehicles as a secondary measure. We then apply a fluid flow analogy to model traffic density and throughput within an arbitrary roundabout system with any number of incoming traffic streams. Due to the distinct differences that separate one-lane roundabouts from two-lane or multi-lane roundabouts, our model considers each lane case separately and determine which case is optimal for a given flow of traffic. The model describes the roundabout system as a non-linear first order partial differential equation relating speed, traffic density, and traffic flow, all of which are subject to physical constraints. The model is able to evaluate how many lanes are needed, and whether traffic light controls are necessary for a given roundabout system. As real-world case study, we apply our model to a roundabout intersection in Alachua Country Florida, and suggest optimal parameters to maximize traffic flow through the roundabout. Applying our model to a variety of roundabout scenarios led us to the following conclusions: traffic lights should not be used at roundabouts; increasing the radius of the roundabout will increase the throughput, but the effect is only significant at high rates (above 0.25 vehicles per second per entering road) of traffic; increasing the number of lanes will always increase throughput, but the benefit only becomes significant when traffic is heavy; increasing radius increases delay for vehicles entering at speeds above 20m/s, while decreases delay for those entering below 20m/s.

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1   Introduction

A traffic circle is a road junction in which vehicles passing through will travel in the same direction (counterclockwise in the US) around a central barrier. According to an extensive study done by the Oregon Department of Transportation, traffic circles are inefficient unless they are designed as roundabouts, which are traffic circles with the rule that traffic entering the circle must yield to traffic already in the circle.[4] For this reason we make a distinction between traffic circles and roundabouts and consider the latter as the most effective version of the former.

By design, roundabouts provide an advantageous alternative to the conventional intersection by reducing the number of traffic flow intersection points (see Fig. 1). If more than four roads come together at one intersection, roundabouts are even more effective when compared to traditional intersections.[4] When used appropriately, roundabouts can significantly increase the efficiency of traffic flow through a multiple-road junction, both in terms of throughput and safety. However, roundabouts have also been known to increase traffic congestion when used with improper methods of traffic control. In order to optimize the flow of traffic through a roundabout, it is our task to develop a model that determines the best means of traffic control for any specific roundabout setting.
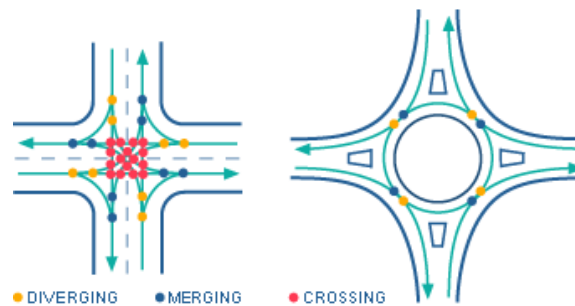
Figure 1: Comparison of conflict points (i.e. vehicle path intersections) between traditional four-way intersections and roundabouts. Because roundabouts have less conflict points, they can often be more effective than standard traffic-light intersections.[9]

1.1   Traffic Circles Versus Roundabouts

There are two main methods of determining which streams of traffic must yield in case of conflict. Either traffic already in the roundabout must yield to traffic entering the roundabout, or traffic entering must yield to the traffic circulating in the roundabout. However, only the latter method is viable[10], as the former easily leads to situations in which no vehicles in the roundabout are capable of moving. According to a study by the Oregon Department of Transportation, since the 1960s, many countries such as the UK used the offside priority rule, in which vehicles entering the roundabout must yield to vehicles already in the roundabout. This method "prevented traffic locking and allowed free-flow movement on the circulating roadway."[4] Consequently, we will primarily consider roundabouts, as they have been demonstrated to be superior to general traffic circles. To make a more thorough distinction between roundabouts and traffic circles, we will consider the characteristics of each in the following table (Table 1).

Table 1: Distinguishing Features of Roundabouts and Traffic Circles [4]

	Modern Roundabout	**Traffic Circle**
Control at Entry	Yield sign for entering vehicles.	Stop, signal or give priority to entering vehicles
Operational Characteristics	Vehicles in the roundabout will have a priority over the entering vehicle.	Allow weaving areas to resolve the conflicted movement
Deflection	Use deflection to control the low speed operation through roundabout.	Some large traffic circles provide straight path for major movement with higher speed.
Parking	No parking is allowed on the circulating roadway.	Some larger traffic circles permit parking within the circulating roadway.
Pedestrian Crossing	No pedestrian activities take place on the central island	Some larger traffic circles provde for pedestrian crossing to, and activities on, the central island.
Turning Movement	All vehicles circulate around the central island.	Mini-traffic circles, left-turning vehicles are expected to pass to the left of the central island.

1.2   Objectives

Our goal is to find the optimal roundabout design for any given intersection. We thus need a metric with which to evaluate roundabout efficiency. In order to create such a metric, **we define the optimal roundabout as the one that allows the greatest possible throughput of vehicles while minimizing the delay experienced by vehicles.** For any given set of traffic conditions, our objective is then to determine roundabout design and parameters that will maximize throughput.

1.3   Advantages of a Macroscopic Model

There are two prominent types of models currently being used to simulate traffic flow, microscopic and macroscopic. Microscopic models focus on individual drivers, and their choices based on their environments. On the other hand, macroscopic models consider large groups of vehicles at once, and model them as fluids, or with other physical analogies. Microscopic models may give a significant amount of information about individual vehicles in a traffic stream, but they rely heavily on extensive data about very specific vehicle interactions.[5] They are thus highly prone to error due to small variation in the many parameters that they require. Additionally, because we are only interested in optimizing global quantities, a microscopic model provides us with extraneous information. In the case of roundabouts, drivers have relatively few choices in the paths that they take, so we can achieve an equal amount of control over the flow of traffic with a macroscopic model. Moreover, the data required by a macroscopic model, such as measures of flow and speed, are easily acquired through routine traffic counting, whereas the data for microscopic models is much more difficult to measure and quantify. Finally, the set of factors that influence throughput are all macroscopic features of the roundabout, and are thus readily incorporated into a macroscopic model.[7]

1.4 Model Considerations

In our comparison between microscopic and macroscopic models, we found that macroscopic models would be the most effective in optimizing the efficiency of a roundabout. In order to maximize throughput at a road junction, it is more appropriate to consider a continuous model that represents an average flow of traffic, as the details of individual vehicles are not relevant to our overall goals. Namely, in order to determine and characterize a method that offers an optimal control of traffic flow in, around, and out of a roundabout circle, we consider the following parameters:

- Radius of the roundabout
- Velocity of traffic moving in the roundabout
- Density of traffic in the roundabout
- Number of lanes in the roundabout
- Type and placement of traffic control signs and signals
- Number of vehicles approaching and entering the roundabout

2 Notation and Definitions

roundabout: A road junction that has at least three input traffic streams and three output traffic streams.

traffic stream: A road that channels vehicles either into or out of a roundabout. A traffic stream can have any number of lanes.

flux (q): The number of vehicles passing a given point over a given interval of time. We will measure this in vehicles per second.

throughput: The amount of vehicles capable of passing through the roundabout per unit time based on the quantity of vehicles trying to enter the roundabout

density (ρ): The number of vehicles per unit of distance. We will measure this in vehicles per meter.

velocity (u): The rate at which the flow of traffic is moving. We will measure this in meters per second.

u_{max}: The maximum velocity that the traffic stream can attain in a given roundabout

ρ_{max}: The maximum density of the traffic stream, at which point all motion halts. Based on data for average car length, we decided to set ρ_{max} equal to 0.25 vehicles per meter.

3 Simplifying Assumptions

- The velocity of a vehicle in traffic is linearly dependent on the density of the traffic at that point. When the density is zero, traffic is unimpeded, so it will flow at the maximum rate possible for the design of the roundabout. When the density is equal to ρ_{max}, traffic will be unable to flow, so the velocity will be zero. This assumption is based on data on traffic density versus speed from literature by Haberman[2] and Edie[6]. We will thus use the following equation for the velocity of the traffic flow:

$$u(x,t) = u_{max}\left(1 - \frac{\rho(x,t)}{\rho_{max}}\right) \tag{1}$$

- The maximum velocity of vehicles in a roundabout is determined by the radius of the roundabout. Using the regression relation in a National Cooperative High Research Program Report,[13] relating maximum speed to radius:

$$u_{max} = 2.41 r^{0.377} \tag{2}$$

- The roads leading away from the roundabout are never clogged. We intend to optimize the efficiency of a single roundabout in isolation since roundabouts are generally placed far away from nearby intersections.[4]
- Vehicles take the optimal path through the roundabout. In a multi-lane roundabout, this means that vehicles will move to the appropriate lane for where they want to exit, and stay there until they reach their exit.
- There are no pedestrians or bicyclists.

4   The Model

4.1   Formula for Describing Traffic Flow

Let $\rho(x, t)$ be the density of the traffic stream at a given point in the roundabout x at a given time t. Let q be the flux of traffic through the point (x, t). We will derive our equation based on the principle of conservation of vehicles, namely, that no vehicles are created or destroyed. Thus, every vehicle that enters any segment of the roundabout must also exit at some later time. From this principle, [2] derives the following equation:

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial q(x, t)}{\partial x} = 0 \tag{3}$$

Where there is an entrance or an exit to the flow of traffic, this equation becomes

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial q(x, t)}{\partial x} = \beta(x, t), \tag{4}$$

where β is the net flux of vehicles entering or exiting the traffic stream at the point (x, t). According to Haberman, "a road is homogeneous such that the vehicle velocity depends on traffic density and not on time and position along the road... flow only depends on the density"[2], thus we generalize to say that at point (x, t), the flux $q(x, t)$ is given by $q = \rho u$. Because we have assumed that the velocity of the traffic stream at any given point is a function only of the constants u_{max} and ρ_{max} and the density at that point, we can write

$$q = \rho u(\rho) \tag{5}$$

where

$$u(\rho) = u_{max} \left(1 - \frac{\rho}{\rho_{max}} \right) \tag{6}$$

Thus, to model the traffic flow, we can use the equation

$$\frac{\partial \rho(x, t)}{\partial t} + u_{max} \frac{\partial \rho(x, t)}{\partial x} - \frac{u_{max}}{\rho_{max}} \frac{\partial \rho^2(x, t)}{\partial x} = \beta(x, t) \tag{7}$$

This first order non-linear partial differential equation thus accounts for traffic entering and leaving the roundabout while subjecting traffic to the physical constraints imposed by the roundabout.

4.2   A Simple Roundabout

The most basic and common form of roundabout is a circular road around a central obstruction with four two-way streets intersecting it. We model this as a circular domain (i.e. one dimensional domain with periodic boundary conditions) on which (7) applies. We set the net flux β equal to zero everywhere except at the points where the incoming and outgoing streams of traffic intersect the lanes of the roundabout. At the incoming lanes, we set β to be a function of the volume of traffic ready to enter at that point and the density of traffic already in the roundabout. In a one-lane roundabout, exiting vehicles are unimpeded, so at the exit points we set the flux β to be the fraction of the vehicles at a given point and time that are expected to exit the roundabout.

4.3   Two-Lane Model

In order to model a two-lane roundabout, we used two separate rings on which to apply our density function. We assumed that vehicles in the left entry lane would go to the inner roundabout lane, and that vehicles in the right entry lane would go the outer roundabout lane. The rate of entry into the outer lane was dependent on only the density of the outer ring at that point, but the rate of entry to the inner lane was determined by the maximum of the density of the outer and inner lanes at the point of entry. The output of the outer lane was unimpeded, but the output of the center lane was slowed by a factor related to the density of the outer lane, because the flow of traffic in the inner lane needs to cross the traffic in the outer lane in order to leave the roundabout.

4.4   Implementation of Our Model

We solved for ρ in (7) numerically in MATLAB using a fourth order Runge-Kutta time-stepping routine. We discretized the spatial domain into 256 points, and assumed periodic boundary conditions (as our domain is a continuous ring). Numerical differentiation was performed spectrally using a Fast Fourier Transform. Using the solution for traffic density ρ, we were able to use incoming flux characteristics (obtained from both actual and simulated data) to determine the outgoing flux of vehicles.

4.5   Results of Our Model

Our first task was to examine how the radius or a roundabout and the number of lanes affect throughput. The results represent the throughput (i.e. the number of vehicles) moving through the roundabouts within a period of three minutes. Note that the flux in Fig. 2 and 3 represents the number of vehicles per second per lane entering the roundabout. The general trend seen for both one-lane (see Fig. 2) and two-lane (see Fig. 3) roundabouts was that the number of vehicles exiting a roundabout increases significantly with the input rate of vehicles. At small input rates (low traffic volume) the radius of the roundabout has little to no effect on the throughput, while at large input rates (0.25 vehicles/s) increasing the radius will increase the throughput.

Based on our model and three-minute simulations of both one and two-lane round-abouts, we found that two-lane roundabouts are able to handle more vehicles in all cases. However, the advantage of having a two-lane roundabout over a one-lane roundabout is not very significant until incoming traffic reaches very high flux. If flux is less than 0.15 vehicles per second, then traffic engineers should consider costs before installing a two-lane roundabout instead of a one-lane.
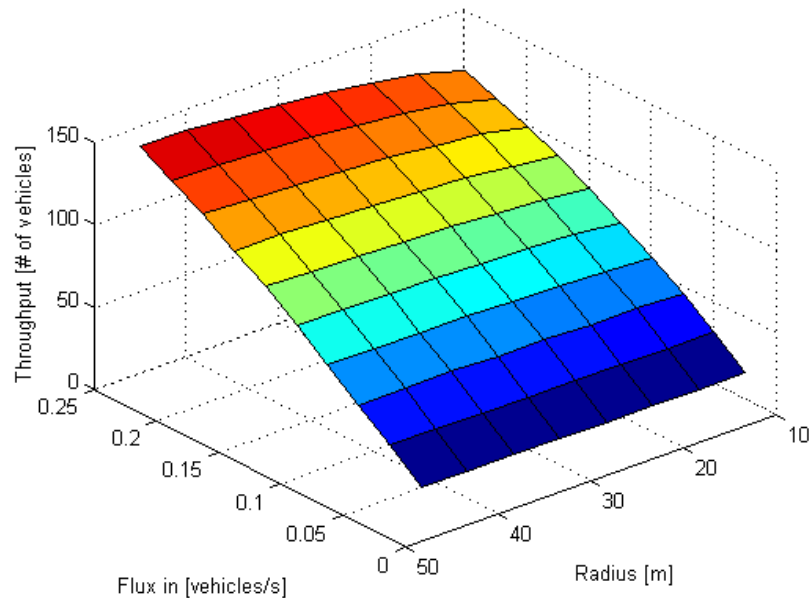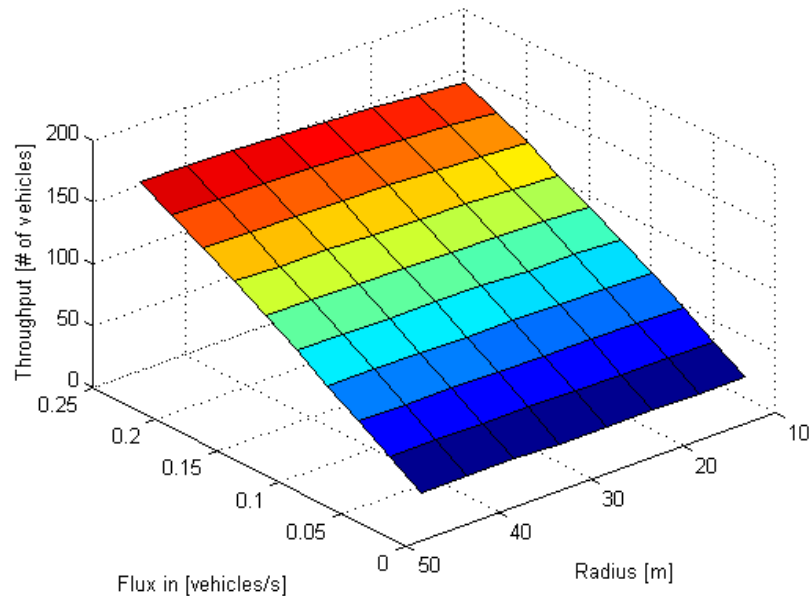
Figure 2: Within a period of 3 minutes, a 10-meter radius roundabout with a single lane can have a maximum of 113 vehicles pass through it. While a 45-meter radius roundabout can have a maximum of 138 vehicles pass through it.

5   Traffic Lights

In addition to considering variable number of lanes, and sizes of roundabouts, our model also takes traffic lights into consideration. Since roundabouts are highly efficient for cases with low traffic [4], in order to model different traffic signal patterns, we consider the worst-case incoming traffic scenario - a high influx of vehicles during rush hour. In order to incorporate traffic lights, instead of modeling flux is constant for a small period of time, we model it as a step function in a number of patterns.

The first pattern we considered is a rotating one in which a traffic signal is green for 30 seconds for one incoming traffic stream, while red for all the rest. The traffic signal then switches so that it is green for another traffic stream for 30 seconds, and red for all the rest, and thus rotates around the circle. The second pattern considered was where traffic streams entering the roundabout on opposite sides would have a green light for periods of 30 seconds. This same pattern was then considered for 60-second cycles. Lastly, we considered a pattern in which a green traffic light signal would favor high-density traffic streams. In each case, the traffic lights impeded and lessened the flow of vehicles through the roundabout. On average, the use of traffic lights with the roundabout resulted in a reduction of throughput of 6.5 vehicles for a 3 minute interval. Based on these findings we conclude that having traffic lights regulate roundabout traffic flow limits roundabout effectiveness and therefore should not be used.

Figure 3: Within a period of 3 minutes, a 10-meter radius roundabout with two lane can have a maximum of 140 cars pass through it, while a 45-meter radius roundabout can have a maximum of 154 vehicles pass through it.

6 Model Versatility Given Irregular Traffic

A major strength of our model is that it has the ability to be adapted to represent any pattern of incoming traffic (see Fig. 6). Moreover it can simulate traffic flow into and out of the roundabout for any time interval. This means that using data available from routine traffic counts our model can be used to determine the optimal roundabout parameters.

7 Using Delay as a Supplementary Metric

In cases where the primary metric, throughput, of two roundabouts is equal, we can use geometric delay as a secondary metric. We define geometric delay to be the difference between the time that it takes a vehicle to traverse a roundabout and the time it would take for a vehicle to go the same distance on a straight road. Let

- V_{ext} be the velocity on a road that enters the roundabout
- V_{int} be the velocity of traffic within the roundabout
- a_{in} be the deceleration of vehicles entering the roundabout
- a_{out} be the acceleration of vehicles leaving the roundabout
- d_r be the distance traveled in the roundabout
- d_1 be the distance from the entrance of the roundabout to the center
- d_2 be the distance from the center of the roundabout to the exit
- d_{dec} be the distance over which deceleration takes place
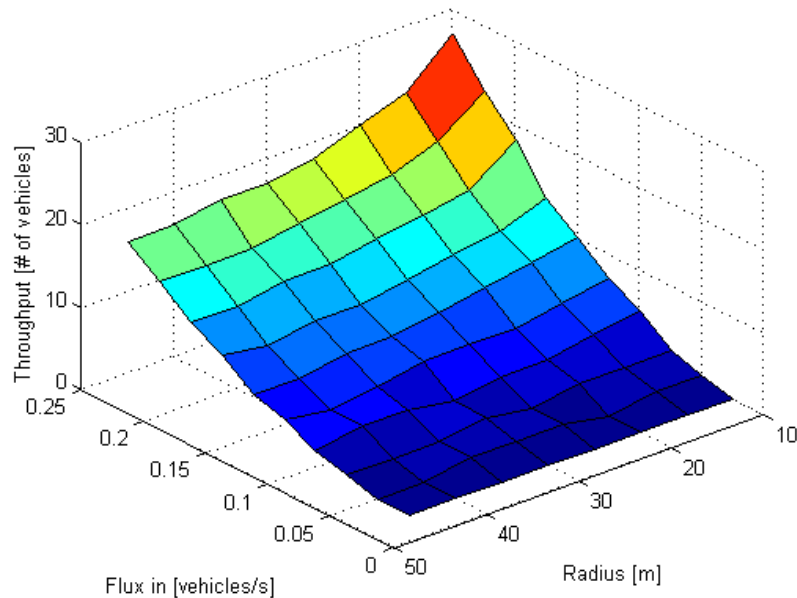
Figure 4: Taking the difference between the one and two lane cases, we find that a two-lane roundabout will make a significant improvement over a single-lane roundabout when the radius is small and the flux of vehicles is large.
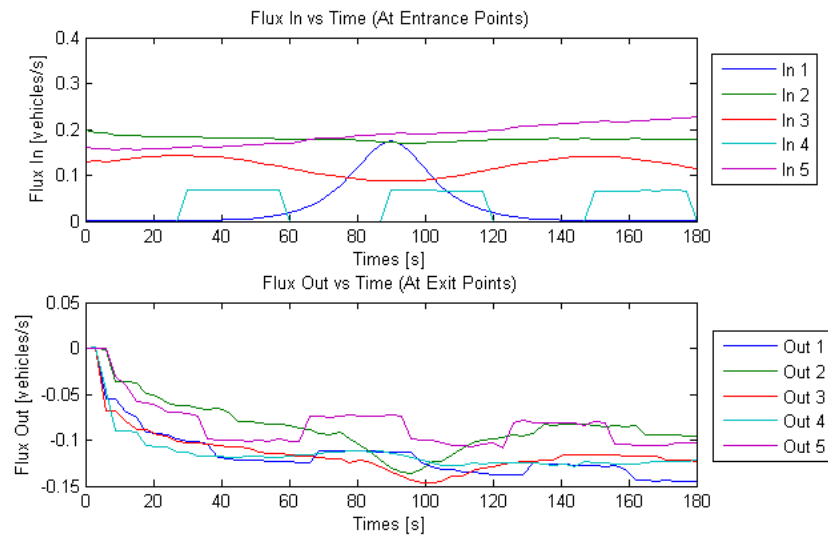
Figure 5: These graphs represent the input and output streams of traffic for a 5-way roundabout for a duration of 180 seconds. Notice the flux is negative for vehicles exiting the roundabout. The versatility of our model is clearly seen in that it can be adapted to represent any type of inflow of traffic.

- d_{acc} be the distance over which acceleration takes place

Then, the geometric delay of a vehicle traveling through the roundabout will be the sum of the time needed to accelerate upon entering, decelerate upon exiting, and to drive through the roundabout (not including the overlapped intervals). The delay is then described by the following relation[4]:

$$\frac{V_{ext} - V_{int}}{a_{in}} + \frac{V_{ext} - V_{int}}{a_{out}} + \frac{d_r}{V_{int}} - \frac{d_1 + d_{dec} + d_2 + d_{acc}}{V_{ext}} \tag{8}$$

The values for V_{int}, d_r, d_1, and d_2 are already encompassed in our model. Using these values and the relations given by the Oregon Department of Transportation, we find that the acceleration of vehicles in the roundabout are described by the following relations:

$$
\begin{aligned}
a_{in} &= 1.11\frac{V_{ext} - V_{int}}{V_{ext}} + .02 \\
a_{out} &= 1.06\frac{V_{ext} - V_{int}}{V_{ext}} + .23 \\
d_{dec} &= \frac{V_{ext}^2 - V_{int}^2}{2a_{in}} \\
d_{acc} &= \frac{V_{ext}^2 - V_{int}^2}{2a_{out}}
\end{aligned}
$$
$$\tag{9}$$

To compare two roundabouts using this delay metric, consider the delay on a path that would otherwise pass straight through the center of each roundabout. In this case $V_{int} = u_{max}$, $d_1 = d_2 = r$, and $d_r = \pi r$. Then assume a common V_{ext} for both roundabouts, and compare the resulting delays. For roundabout that have the same throughput, the one with shorter geometric delay will be designated as optimal.
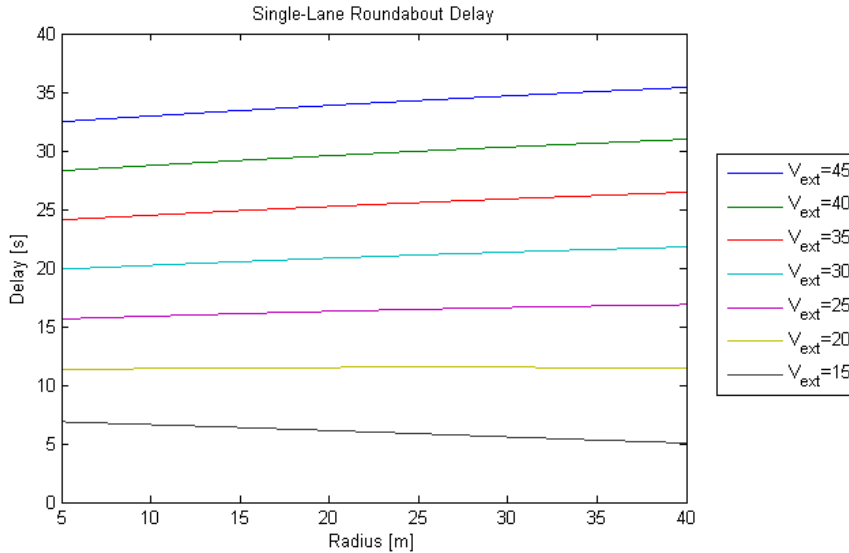
Figure 6: Graph of geometric delay for roundabouts of radii between 5 and 40 meters with external velocities between 15 and 45 m/s (33 and 101 mph).

8   Case Study: Applying Our Model to a Real-World Scenario

In 2006, Florida's Alachua County Public Works Department performed a roundabout justification study on the T-intersection on NW 143rd Street (CR 241) and NW 39th Avenue, a few miles outside of Gainesville, Florida.[12] The goal of the study was to determine if a roundabout was justified at the intersection, and to make a recommendation on future traffic control at that location. We apply our model to the incoming vehicle flux data given in the report and suggest an optimal roundabout design for the intersection.

8.1   Current Conditions

NW 143rd Street is a two lane highway in the vicinity of the intersection, with a posted speed limit of 45 mph. It has a left turn lane on the southbound approach, and a right turn lane on the northbound approach. NW 39th Street dead ends into NW 143rd Street, and has both left and right turn lanes on the westbound approach. Approximately 6% of the vehicles using the intersection are heavy trucks.

Figure 7: The intersection NW 143rd Street and NW 39th Street

8.2   Data Available

The Alachua County Public Works Department performed a study in 2004 that provides vehicle classification counts for each approach, a turning movement count in each direction, and a speed study on NW 143rd Street. The car count data is available at every 15 minute interval from 5:15 AM to 7:30 PM.

8.3   Application of our Model

We tested our model under average and peak loads during both the morning and afternoon time periods, with the data taken directly from the roundabout justification study. Based on the number of cars entering the roundabout from a given direction, we calculated the flow of cars per second entering. Similarly, based on the desired destinations of the cars from the data collected by the Alachua County Public Works Department[12], we calculated the proportion of cars that would exit at each point in the roundabout. We

then varied the number of lanes, the radius, and the stop light patterns (i.e. duration of light times) and ran simulations over a 15 minute time interval to determine which parameters maximized the throughput of cars.

8.4 Conclusions

For the rates of cars wanting to enter the roundabout in this problem, our model finds that the addition of traffic lights will generally either decrease or have little-to-no effect on throughput. Therefore, we conclude that

- A traffic light system would not be appropriate
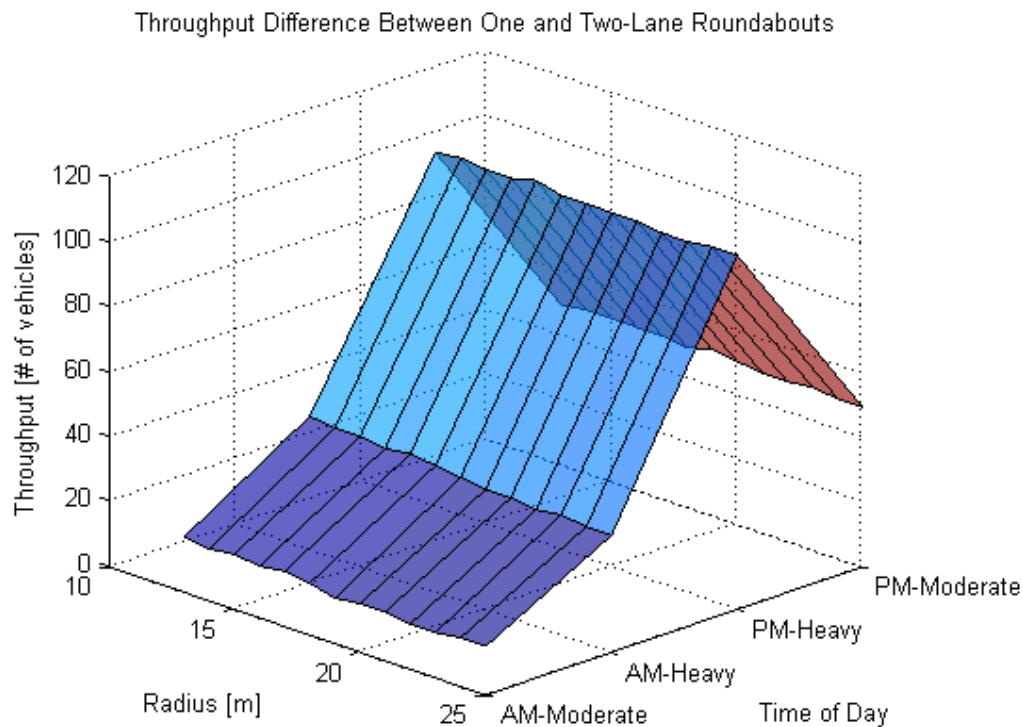
Figure 8: This graph indicates that for heavy evening rush hour traffic, a two-lane roundabout would be optimal.

Figure 8 shows the difference in throughput between a one-lane and two-lane round-about over a 15 minute time-span. In the morning hours, the advantages of using a two-lane roundabout over a one-lane are much less pronounced. However, during the afternoon rush hour, clearly the the two-lane roundabout is a much more efficient design in maximizing throughput. Taking this into account, we propose that

- If the budget allows, a two-lane roundabout would be optimal

Our model indicates that once the radius is above 18m, there is very little variation in throughput in both the one and two lane models. Furthermore, 18m is large enough that most large semi-trucks (according to the Surface Transportation Assistance Act

the standard semi-truck length is 20.73 meters[4]) can fit through it without a problem. Therefore, we suggest

- A radius of 18m (or more) should be used

Finally we used our model to determine the delay associated with the geometry of the roundabout (see Section 7). For the case of a two lane roundabout, the geometric delay as found to be 11.0 seconds per vehicle, while a one-lane roundabout had a geometric delay of 11.9 seconds per vehicle. For both of these cases the model was evaluated without traffic lights. With traffic lights the delay per vehicle was found to be 18.5 seconds and 19 seconds for a two-lane and one-lane case respectively. This approximations fell in line with Alachua County Public Works Department's estimation of 10 to 20 second delays for roundabouts.

For the parameters we tested it was found that the output flux for a one-lane roundabout could reach up to 1200 vehicles per hour. This falls in lane with US Department of Transportation Roundabout Guidelines[8], which state that a throughput of more than 1400 vehicles per hour is not possible in a single-lane roundabout [8]. Since the Alachua County roundabout we considered is an intersection of three roads as opposed to the conventional four, it makes sense for the maximum flux to be lower.

9   Evaluating Our Model

Our model gives an accurate picture of the efficiency of roundabouts, based on control methods, and design parameters of a roundabout. The continuous model that we use is very suitable for evaluating traffic patterns and flows, but is not applicable to more localized phenomena. Our model does not take into account the presence of pedestrians and bicyclists, or the decisions of individual drivers. However, none of these factors are relevant to the evaluation of the efficiency of a roundabout. A further study that wishes to evaluate safety or other features of roundabouts would likely need to take these additional factors into consideration.

10   Conclusion

There are a number of different approaches to modeling traffic flow in a roundabout. Given the goal of maximizing throughput over a given time interval, we developed a robust macroscopic model for simulating the flow of traffic in a roundabout based on the principle of conservation of vehicles. Our model accepts and considers a range of parameters and inputs commonly encountered in real-world traffic scenarios (including traffic signals), and compares the throughput in each case to determine an optimal configuration of parameters, taking into account the delay experienced by a car given the geometry of the roundabout. As a case study, we design a roundabout for an intersection in Alachua County, Florida, and demonstrate that our model can be applied to a real-world scenario with tangible and useful results. In the next section we provide a technical summary for traffic engineers who intend to design a roundabout.

11   Technical Summary for Traffic Engineers

11.1   Design

In order to determine the optimal roundabout design for an intersection, first determine the expected traffic flow for the intersection. If traffic flows of more than 0.6 vehicles per second are expected to enter the roundabout, a two-lane roundabout should be used. Otherwise, a one-lane roundabout will be preferable, as a two-lane roundabout would be more expensive and only marginally more effective than a one-lane roundabout. Increasing the radius of the roundabout will increase its efficiency, but will also increase the per-vehicle delay. We recommend building the roundabout with the largest radius less than 50 meters that the space and budget of the intersection will allow.

11.2   Control

The primary means of controlling the traffic flow in the roundabout will be with signs. Yield signs should be posted at each entrance to the roundabout, to ensure that cars entering will yield to cars already circulating. A sign indicating the reduction of the speed limit in the roundabout should also be posted (see Table 2). The intersection

Table 2: Posted speed limits based on radius

Radius (m)	Velocity (m/s) (mph)
5	4.4 (10)
10	5.7 (10)
15	6.7 (15)
20	7.5 (15)
25	8.1 (15)
30	8.7 (20)
35	9.2 (20)
40	9.7 (20)
45	10.1 (20)
50	10.3 (25)

should be well lit at all times, to ensure that the structure of the roundabout is obvious to drivers.

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