Math 125

Midterm 2 (February 27, 2020)

NAME: ______________________ **Section:** _____________

- Time: you have **80 minutes**.
- Please show all work and justify your answers. The final answers must be "reasonably" simplified. For example, a rational number must be given in the form $\frac{a}{b}$ for some integers a and b, but it is ok to have expressions like $\ln 3$ or e^4 in your final answer.
- You are allowed to use calculator (Model TI-30X IIS only) and one *handwritten* (with your own handwriting) 8.5 x 11 inch sheet of notes. Writing allowed on both sides.
- Have your *Husky Card* visible on the desk beside you.
- You may use both sides of the paper.
- Make sure you have **9 pages** and **6 problems** before starting the exam.

Academic integrity is expected of all students at all times. Understanding this, I declare I shall not give, use, or receive unauthorized aid.

SIGNATURE: ______________________

Problem 1: ___ / 20
Problem 2: ___ / 20
Problem 3: ___ / 20
Problem 4: ___ / 20
Problem 5: ___ / 20
Problem 6: ___ / 20

Total: ___ / 120

Problem 1: Evaluate the following integrals:

(a)

$$\int \frac{x^4}{\sqrt{(x^2+1)^7}}\,dx$$

(b)

$$\int_0^1 \tan^{-1} x\,dx$$

Recall: $\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{x^2+1}$.

Problem 2: Compute the integral

$$\int \frac{x^2 + 1}{x^3 + x^2}\, dx$$

by the *method of partial fractions.*

Problem 3: Determine, with justification, the *convergence* or *divergence* of each of the following improper integrals.

(a)

$$\int_{-\infty}^{+\infty} xe^{-x^2}\ dx$$

(b)

$$\int_{1}^{+\infty} \frac{x}{\sqrt{x+x^6}}\ dx$$

Problem 4: Consider the function

$$f(x) = x^4 .$$

(a) Let M be the average value of f on $[0, 3]$. Compute M.

(b) Find a value of c in $[0, 3]$ such that $f(c) = M$.

(c) The average value of a function g over $[0, x]$ is equal to x^2 for all x. Determine $g(x)$.

Problem 5: Let

$$f(x) = \frac{1}{4}x^2 - \frac{1}{2}\ln x \ .$$

Find the *arc length* of the curve $y = f(x)$ over the interval $[1, e]$.

Problem 6: Let

$$f(x) = e^{-\sqrt{24}\ x} \ .$$

Find N such that M_N approximates the integral

$$\int_0^{10} f(x)dx$$

with an error of at most 10^{-3}.

Hint: Here M_N denotes the N^{th} *midpoint approximation* to the integral. We have the error bound formula:

$$\left| \int_a^b f(x)dx - M_N \right| \leq K \frac{(b-a)^3}{24N^2} \ ,$$

where K is any real number such that $|f''(x)| \leq K$ for all $a \leq x \leq b$.

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