1. **[8 points]** The speedometer readings on a car, in miles per hour, were observed at 1-minute intervals during an 8 minute trip and recorded in the following chart.

	t (min)	v (mi/h)		t (min)	v (mi/h)
t_0:	0	40	t_3	6	56
	1	42		7	57
t_1:	2	45	t_4	8	57
	3	49			
t_2:	4	52			
	5	54			

Use Simpson's Rule **with** $n = 4$ **subintervals** to estimate the distance traveled by the car during this 8 minute trip. Show your work and include units in your final answer.
(*speaking of units, recall that there are 60 minutes in an hour*)

$$\Delta d = \int_0^{8/60 \text{ hrs}} v(t)\,dt \cong S_4 = ?$$

$$\Delta t = 2 \text{ min} = \frac{1}{30} \text{ hrs.}$$

$$S_4 = \frac{\Delta t}{3}\left(v(t_0) + 4v(t_1) + 2v(t_2) + 4v(t_3) + v(t_4) \right)$$

$$= \underbrace{\frac{(1/30)}{3}}_{\text{hrs}} \Big[\underbrace{40 + 4(45) + 2(52) + 4(56)}_{\text{miles/hr.}} + 57 \Big]$$

$$= \frac{1}{90}\,[605] = \boxed{\frac{121}{18}} \cong \boxed{6.72 \text{ miles}}.$$

2. [**8 points**] Consider the improper integral: $\int_1^e \frac{1}{x(\ln(x))^2}\,dx$. If it converges, evaluate it. If it diverges, say so and show why.

Show your work, and include the limits you compute.

$$\int_1^e \frac{1}{x(\ln x)^2}\,dx = \int_{\ln 1 = 0}^{\ln e = 1} \frac{1}{u^2}\,du$$

$$\boxed{\begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \end{array}}$$

So we need to compute the improper integral:

$$\int_0^1 \frac{1}{u^2}\,du = \lim_{t\to 0^+} \int_t^1 \frac{1}{u^2}\,du$$

$$= \lim_{t\to 0^+} \left(-\frac{1}{u}\Big|_t^1 \right)$$

$$= \lim_{t\to 0^+} \left(-1 + \frac{1}{t} \right)$$

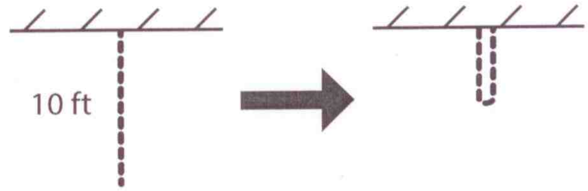
$$= -1 + \lim_{t\to 0^+} \left(\frac{1}{t} \right)$$

$$= \boxed{+\infty}$$

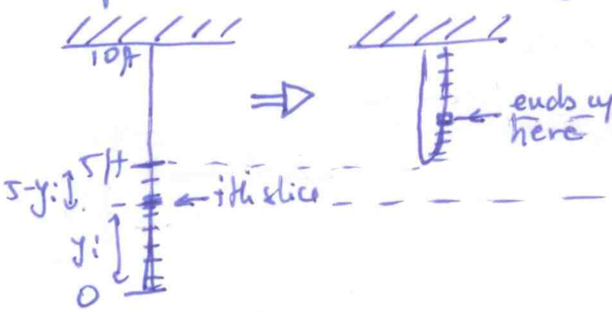
Hence the $\boxed{\text{integral diverges}}$.

3. [**10 points**] A 10-ft chain weighs 25 lbs and hangs from a ceiling. Find the work done in lifting the lower end of the chain to the ceiling so that it's level with the upper end.

Show your work and how you set up any integrals you compute.



Method I (as a "Case 2"): Slice the lower half of the chain (part being lifted) into n "slices" of length Δy each, and calculate the work required to lift each slice. With origin at the bottom of the chain:



For the ith "slice":

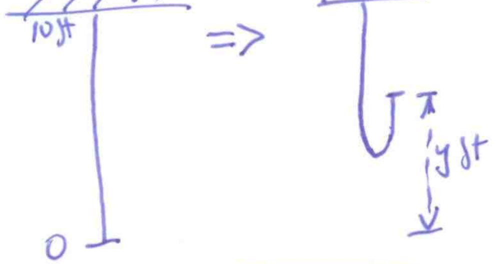
$d_i = 2(5-y_i)$ ft (distance to lift the ith slice)

$F_i = \rho\,\Delta y = \left(\frac{25 \text{ lbs}}{10 \text{ ft}}\right)\Delta y \text{ ft} = 2.5\,\Delta y$ lbs.

$\Rightarrow W_i = F_i d_i = (2.5)(2)(5-y_i)\,\Delta y$ ft-lbs.

$$\boxed{W = \lim_{n\to\infty}\sum_{i=1}^{n} W_i = \int_0^5 5(5-y)\,dy} = \left(25y - 5\frac{y^2}{2}\right)\Big|_0^5 = \boxed{\frac{125}{2} = 62.5 \text{ ft-lb}}$$

Method II (as a "Case 1") Express the weight (force) of the portion we are lifting as a function of the distance y, and take $W = \int_a^b F(y)\,dy$.



When we lifted the end y feet ($0 \le y \le 10$ ft), we are lifting a length of $\frac{y}{2}$ ft

so $F(y) = \left(\frac{25}{10} \text{ lbs/ft}\right)\left(\frac{y}{2} \text{ ft}\right) = \frac{2.5y}{2}$

$$\boxed{W = \int_0^{10} F(y)\,dy = \int_0^{10} 2.5\left(\frac{y}{2}\right)dy} = 2.5\frac{y^2}{4}\Big|_0^{10} = \boxed{\frac{250}{4} = \frac{125}{2} = 62.5 \text{ ft-lb.}}$$

4. Compute the following definite integrals. Simplify, but leave your answers in exact form.

(a) **[6 points]** $\int_0^{\pi/3} \tan^4(\theta)\, d\theta$ $= \int_0^{\pi/3} (\sec^2\theta - 1)^2 \, d\theta$ $\boxed{\text{using: } \tan^2\theta = \sec^2\theta - 1}$

$= \int_0^{\pi/3} (\sec^4\theta - 2\sec^2\theta + 1)\, d\theta$

$= \underbrace{\int_0^{\pi/3} (1+\tan^2\theta)\sec^2\theta\, d\theta}_{\substack{u = \tan\theta \\ du = \sec^2\theta\, d\theta}} - 2\int_0^{\pi/3} \sec^2\theta + \int_0^{\pi/3} 1\, d\theta.$

$= \int_{\tan 0 = 0}^{\tan\frac{\pi}{3} = \sqrt{3}} (1+u^2)\, du - 2\tan\theta \Big|_0^{\pi/3} + \theta\Big|_0^{\pi/3}$

$= \left(u + \frac{u^3}{3}\right)\Big|_0^{\sqrt{3}} - 2(\tan \pi/3 - \tan 0) + \left(\frac{\pi}{3} - 0\right)$

$= \left(\sqrt{3} + \frac{3\sqrt{3}}{3}\right) - (0) - 2(\sqrt{3} - 0) + \frac{\pi}{3} = 2\sqrt{3} - 2\sqrt{3} + \frac{\pi}{3} = \boxed{\frac{\pi}{3}}$

(b) **[6 points]** $\int_0^{1/2} \arcsin(x)\, dx$

Integration By Parts:

$u = \arcsin(x) \quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\, dx \quad v = x$

$\int_0^{\frac{1}{2}} \arcsin(x) = x \arcsin x \Big|_0^{1/2} - \underbrace{\int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\, dx}_{\substack{u = 1-x^2 \\ du = -2x\,dx}}$

$= \left(\frac{1}{2}\arcsin\frac{1}{2} - 0\right) - \int_1^{3/4} \frac{1}{\sqrt{u}}\left(-\frac{1}{2}\right) du$

$= \frac{1}{2}\cdot\frac{\pi}{6} + \left(\frac{1}{2}\right)\frac{u^{1/2}}{(1/2)}\Big|_1^{3/4}$

$= \frac{\pi}{12} + \sqrt{\frac{3}{4}} - \sqrt{1}$

$= \boxed{\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1}$

OR: $u = \arcsin(x) \Rightarrow x = \sin(u)$

$du = \frac{1}{\sqrt{1-x^2}}\, dx$

$\Rightarrow dx = \sqrt{1-x^2}\, du = \sqrt{1-\sin^2(u)}\, du$

$= \sqrt{\cos^2 u}\, du$

$\int_0^{1/2} \arcsin(x)\, dx = \int_0^{\pi/6} u \sqrt{\cos^2 u}\, du$

$= \int_0^{\pi/6} u \cos u\, du$

IBP: $w = u \quad dv = \cos u\, du$

$dw = du \quad v = \sin u$

$= u \sin u \Big|_0^{\pi/6} - \int_0^{\pi/6} \sin u\, du$

$= \left(\frac{\pi}{6}\cdot\frac{1}{2} - 0\right) + \cos u\Big|_0^{\pi/6}$

$= \frac{\pi}{12} + \cos\frac{\pi}{6} - \cos 0$

$= \boxed{\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1}$