1. A producer is figuring out how much to charge for tickets to a show. If she charges \$0 per ticket, she will make \$0. If she charges \$5 per ticket, she will make \$1775. If she charges \$15 per ticket, she will make \$975.

   If the amount of money she makes is a quadratic function of the ticket price, what is the maximum possible amount of money she can make from the sale of the tickets?

x = price per ticket

f(x) = how much she makes. $= ax^2 + bx + c$

Know: 1) $f(0) = \$0 \Rightarrow a(0)^2 + b(0) + c = 0 \Rightarrow \boxed{c = 0}$

2) $f(5) = 1775 \Rightarrow \boxed{25a + 5b + \cancel{c} = 1775}$

3) $f(15) = 975 \Rightarrow \boxed{225a + 15b + \cancel{c} = 975}$

After we eliminate c = 0, we get 2 equations in a & b:

$$\begin{cases} 25a + 5b = 1775 \\ 225a + 15b = 975 \end{cases}$$

Divide both by 5:

$$\begin{cases} 5a + b = 355 \Rightarrow \boxed{b = 355 - 5a} \\ 45a + 3b = 195 \end{cases}$$

$\boxed{b = 500}$

$45a + 3(355 - 5a) = 195$

$45a + 1065 - 15a = 195$

$30a = -870$

$a = \frac{-87}{3} \Rightarrow \boxed{a = -29}$

So $\boxed{f(x) = -29x^2 + 500x}$ This is a quadratic, whose graph is concave-down $(a = -29 < 0)$. Hence it's maximal at vertex: $x = -\frac{b}{2a} = -\frac{500}{2(-29)} = \frac{250}{29} \cong 8.620689\ldots$

That is, she should charge $x \cong \$8.62$ /ticket.

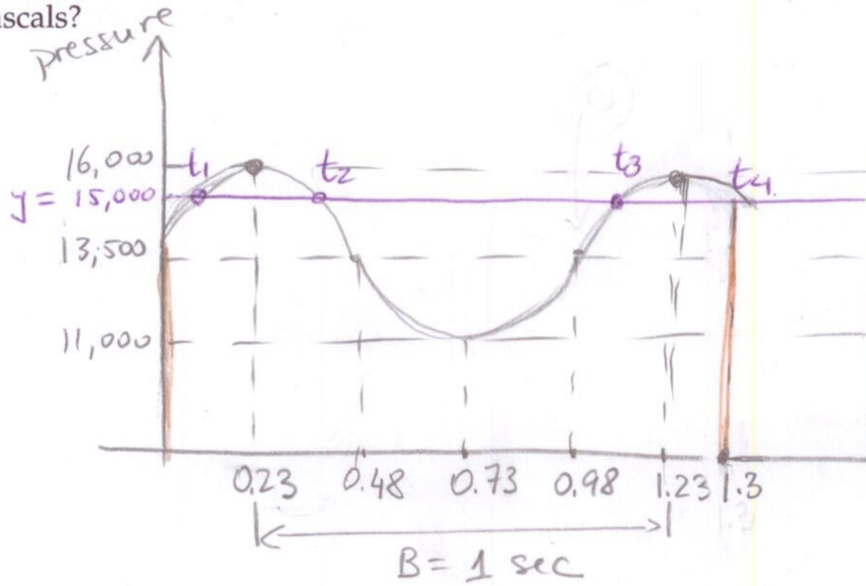
to make a max. of:

$f(8.62) = -29(8.62)^2 + 500(8.62) = \boxed{\$2155.17}$

2. The pressure inside an artery is varying sinusoidally. With a heart beat at a rate of 60 beats per minute, the period of this sinusoidal function is 1 second. The maximum pressure is 16000 pascals, and the minimum pressure is 11000 pascals.

   At time $t = 0$, you begin to measure the pressure. A maximum is attained for the first time at $t = 0.23$ seconds.

   Between $t = 0$ and $t = 1.3$, how much time (in seconds) is the pressure above 15000 pascals?



$A = \frac{16{,}000 - 11{,}000}{2} = 2{,}500$

$D = \frac{16{,}000 + 11{,}000}{2} = 13{,}500$

$B = 1$ (given)

$c = x_{max} - B/4$

$= 0.23 - 0.25 = -0.02$

OR $1.23 - 0.25 = 0.98$

OR ...

$P(t) = 2{,}500 \sin\left(\frac{2\pi}{1}\underbrace{(t - 0.98)}_{\text{OR: } (t+0.02)}\right) + 13{,}500.$

$15{,}000 = 2{,}500 \sin(2\pi(t - 0.98)) + 13{,}500$

$\sin(2\pi(t - 0.98)) = \frac{15{,}000 - 13{,}500}{2{,}500} = 0.6$

PS: $2\pi(t - 0.98) = \sin^{-1}(0.6)$

$t = \frac{\sin^{-1}(0.6)}{2\pi} + 0.98$

$\approx \boxed{1.08241638 = t_3}$ on graph

subtracting 1 period $B = 1$

$\boxed{t_1 = 0.08241638}$

SS: $2\pi(t - 0.98) = \pi - \sin^{-1}(0.6)$

$t = \frac{\pi - \sin^{-1}(0.6)}{2\pi} + 0.98$

$\approx \boxed{1.37758362\ldots = t_4}$

$\boxed{t_2 = t_4 - B \approx 0.3775836\ldots}$

note that t_4 is > 1.3 so we'll only go to 1.3.

The pressure is above the purple line of 15,000 Pa from t_1 to t_2 and from t_3 to 1.3

so total time $= (t_2 - t_1) + (1.3 - t_3)$

$\approx (0.295167) + (0.2175836) \approx$ 0.51275 sec