Math 120 Final

Winter 2005, Saturday March 12th.

Section: ____________ **Name:** ____________________________________

Student Number:________________________

Instructions:

- You have 3 hours for this exam.
- One 8.5 by 11 inch page of handwritten notes (front and back) is allowed.
- Write your name and section on your page of notes. Turn your page of notes in with your exam.
- Calculators (scientific or graphing) are allowed.
- If you need more space, use the backside of a page. Indicate that you have done so.
- You must show your work for full credit. Answers obtained by guessing or reading a numerical solution from a graph on your calculator when an algebraic method is available do not receive full credit.
- Clearly indicate your answers (e.g., by boxing them).
- There are 8 problems, and 9 pages to the exam. Check to make sure your exam is complete.

Problem	Total Points	Points
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
Total	96	

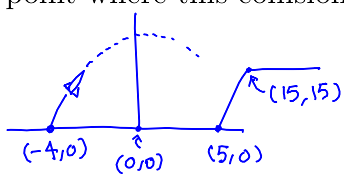
Problem 1. Mary is jogging on a straight line from the point $(-2, 10)$ to the point $(14, -9)$. There is a donut shop located at the point $(4, 7)$. (Assume units are in miles.)

(a) [9 pts] What are Mary's coordinates when she is closest to the donut shop?

(b) [3 pts] If Mary get's within 2.7 miles of a donut shop, she will stop her jog and get a donut! Does Mary stop her jog and get a donut? (Show work for full credit.)

Problem 2. Joey is trying to fire his model rocket over an embankment. The rocket starts at the point $(-4, 0)$ and follows the path $y = 20 + 3x - \frac{1}{2}x^2$. The embankment starts at the point $(5, 0)$ and follows a straight line to the point $(15, 15)$, where the embankment ends. (Assume units are in feet.)

(a) [6 pts] Determine if the rocket collides with the embankment, and if it does, find the point where this collision occurs.



(b) [4 pts] The rocket launcher has a control dial on it. When the control dial is set to α, the path of the rocket is

$$y = -\frac{1}{2}\alpha(x+4)(x-10\alpha).$$

Joey would like the x coordinate for the vertex of the rocket path to occur at $x = 15$ (where the embankment ends). What should α be set to in order for this to happen?

(c) [2 pts] With this value of α, find the height of the rocket at its vertex.

Problem 3 Let u(t) be the basic step function:

$$\mathrm{u(t)} = \begin{cases} 0 \text{ if } t < 0 \\ 1 \text{ if } 0 \leq t \leq 1 \\ 0 \text{ if } 1 < t \end{cases}$$

a) [4 pts] Find the multipart rule for $f(t) = t^2\mathrm{u}(\frac{1}{4}(\mathrm{t}+1))$

b) [4 pts] Find the multipart rule for $g(t) = f(t) + t\mathrm{u(t)} + 1$.

c) [2 pts] What is $g(\frac{1}{2})$?

d) [2 pts] What is $g(3)$?

Problem 4. Suppose that

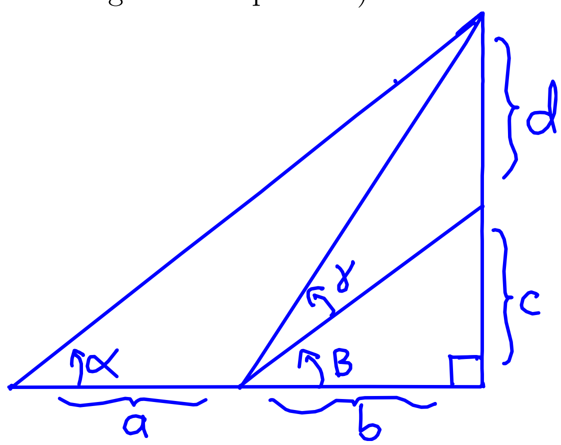
$$f(x) = x + \sqrt{x^2 + 1}$$

and that the domain for $f(x)$ is $x \geq 2$.

(a) [8 pts] Find $f^{-1}(x)$.

(b) [4 pts] What is the domain of $f^{-1}(x)$?

Problem 5. In the following picture, suppose that $a = 10$ feet, and that the angle $\alpha = 32°$, the angle $\gamma = 22°$ and the angle $\beta = 42°$. Find the length of d. (Note the right angle at the lower right of the picture.)

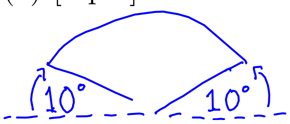


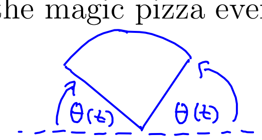
Problem 6. Suppose that $f(t)$ is a sinusoidal function of time (in seconds), which oscillates between a minimum value of .5 and a maximum value of 2.5. When $t = 1$, $f(t)$ is at its minimum value. Between time $t = 1$ and time $t = 4$, $f(t)$ reaches its maximum value exactly twice. At time $t = 4$, $f(t)$ is at its minimum value.

(a) [6 pts] Find $f(t)$.

(b) [6 pts] Suppose that the voltage output of an electrical circuit is $V(t) = e^{f(t)}$. Between $t = 1$ and $t = 4$, what percentage of the time is $V(t) \geq 7$?

Problem 7. Marvin the Magician is entertaining his guests with his magic pizza trick, he starts with half a pizza minus $10°$ from either end, as in the first figure. Then magically, the pizza shrinks, as the amount taken away on both ends is given by $\theta(t) = \frac{45t+10}{t+1}$ where θ is in degrees and t is in seconds (as in the second figure). The radius of the pizza is 10 inches.
(a) [6 pts] Is the area of the magic pizza ever 90 in^2? If so, find this time.

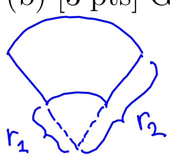




For Marvin's next trick, the angle taken away on either side is still given by $\theta(t)$ as above, but at the same time the inner radius r_1, and outer radius r_2, as in the picture below, are changing as well. They are changing according to

$$r_1(t) = \frac{4t}{t+1} \qquad r_2(t) = \frac{8t+10}{t+1},$$

(b) [3 pts] Give the area of the magic pizza as a function of t. (You do not need to simplify.)



(c) [3 pts] If one were to watch Marvin's magic pizza forever, with the changing $\theta(t), r_1(t)$ and $r_2(t)$, the area of the magic pizza would approach a certain value. What is this value?

Problem 8. Bobby the physicist has been studying strange signals coming from outer space. The signal strength S tends to be very large, with values ranging up to 10^{21}. Bobby is using a logarithmic scale to describe these signals. Taking S_0 to be a reference value, he has defined the signal γ value to be

$$\gamma = 4\log_{10}\left(\frac{S}{S_0}\right)$$

(a) [6 pts] Bobby is studying two signals with signal strengths S_1 and S_2. S_2 is twice as large as S_1. If γ_1 and γ_2 are the γ values for S_1 and S_2, what is $\gamma_2 - \gamma_1$?

(b) [6 pts] Sheila the scientist has also been studying the signals, however she is using a different logarithmic scale. She has defined the β value of a signal S to be

$$\beta = \log_b\left(\frac{S}{S_0}\right).$$

for a certain base b and reference value S_0. It turns out that a signal strength of 10^{21} has a signal β value of 100, and a signal strength of 10^9 has a signal β value of $33\frac{1}{3}$. What are S_0 and b? If possible, give an exact value for S_0.