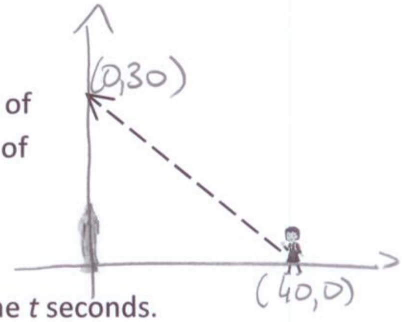


Problem 1 (16 pts) Linda is walking in a straight line from a point 40 feet due east of a statue, to a point 30 feet due north of the statue. She walks at a constant speed of 2 feet per second.

a) Impose a coordinate system with the origin at the statue and determine the parametric equations giving the x and y-coordinates for Linda's position at time t seconds.

From $(40,0)$ to $(0,30)$, Linda travels $\sqrt{(30)^2+(40)^2} = \boxed{50 \text{ feet}}$

At 2 ft/sec, it takes her $\Delta t = \frac{d}{v} = \frac{50 \text{ feet}}{2 \text{ ft/sec}} = \boxed{25 \text{ seconds}}$

Her coordinates at t seconds are:

$$\boxed{\begin{aligned} x(t) &= 40 - 1.6t \\ y(t) &= 1.2t \end{aligned}}$$

since $x(0)=40$, and $V_x = \frac{\Delta x}{\Delta t} = \frac{0-40}{25} = \frac{-8}{5} = -1.6$

$y(0)=0$, $V_y = \frac{\Delta y}{\Delta t} = \frac{30}{25} = \frac{6}{5} = 1.2$

b) Find all the times when Linda is at a distance of 25 feet from the statue.

Distance Linda to statue at t seconds is: $d(t) = \sqrt{(x(t)-0)^2 + (y(t)-0)^2}$

so $25 = \sqrt{(40-1.6t)^2 + (1.2t)^2}$, which simplifies to

$25 = \sqrt{4t^2 - 128t + 1600}$

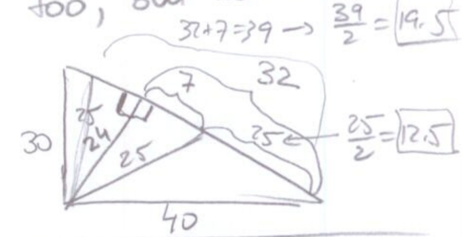
Squaring and simplifying:

$4t^2 - 128t + 975 = 0$

Quadratic formula $t = \frac{128 \pm \sqrt{128^2 - 4(4)(975)}}{8}$

so $d(t) = 25$ at $\boxed{t = 12.5 \ \& \ 19.5 \text{ sec}}$

Note: this can be solved geometrically too, but harder.



c) Find Linda's position (x, y) when she is closest to the statue.

METHOD I: VERTEX OF PARABOLA

$d^2 = 4t^2 - 128t + 1600$

concave-up ⇒ min. at vertex

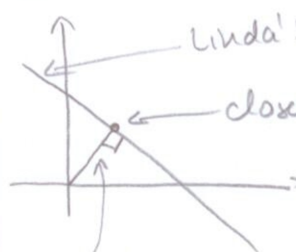
$t = -\frac{(-128)}{2(4)} = 16 \text{ sec}$

$x(16) = \dots = 14.4$

$y(16) = \dots = 19.2$

position $\boxed{(14.4, 19.2) \text{ feet}}$

METHOD II: PERPENDICULAR to PATH.



Linda's path: $y = -\frac{3}{4}x + 30$

closest to statue at intersection with line through origin perp. to path

$y = \frac{4}{3}x$ and $y = -3/4\, x + 30$

$\Rightarrow \frac{4}{3}x = -3/4\, x + 30 \Rightarrow x = 14.4$

$14.4 = x = 40 - 1.6t \Rightarrow t = 16$

$\Rightarrow y(16) = \dots = 19.2$

Problem 2 (10 pts) Solve the following two equations. Show all steps, and box your final answers.

a) $7x + |2x - 5| = 4$

$$7x + |2x-5| = \begin{cases} 7x + (2x-5) & \text{if } 2x-5 \geq 0 \\ & \quad 2x \geq 5 \\ & \quad x \geq 5/2 \\ 7x - (2x-5) & \text{if } x \leq 5/2 \end{cases} = \begin{cases} 9x-5 & \text{if } x \geq 5/2 \\ 5x+5 & \text{if } x \leq 5/2 \end{cases}$$

Case 1: If $x \geq 5/2$ the equation is $9x - 5 = 4$

$9x = 9$

$x = 1 \leftarrow$ not a sol. because it's not $\geq 5/2$

Case 2: If $x \leq 5/2$ then we solve: $5x+5=4$

$5x = -1$

$x = -1/5 \leftarrow$ it is $\leq 5/2$ so it is a sol.

$\boxed{\text{Sol: } x = -1/5}$

b) $ln\left(\dfrac{2x}{x-5}\right) = 3$

$$e^{\ln\left(\frac{2x}{x-5}\right)} = e^3$$

$$\frac{2x}{x-5} = e^3$$

$$2x = e^3(x-5)$$

$$2x - e^3x = -5e^3$$

$$\boxed{x = \frac{-5e^3}{2-e^3} = \frac{5e^3}{e^3-2} \cong 5.55}$$

Problem 3 (12 points)

The population of Arcadia increases by 8% every 10 years. The population of Brom triples every 120 years.

The two cities had equal populations of 10,000 residents each in the year 2000.

In what year will the city of Brom have twice as many residents as the city of Arcadia?

Arcadia: $A(t) = A(0)\, a^t = 10{,}000\, a^t$ $\quad \boxed{t = \text{years after year } 2000}$

$t = 10$ years: $A(10) = 10{,}000 + 0.08(10{,}000)$
$= 10{,}800$

$10{,}800 = A(10) = 10{,}000\, a^{10}$

$1.08 = a^{10} \Rightarrow a = \sqrt[10]{1.08}$

so $\boxed{A(t) = 10{,}000\left(\sqrt[10]{1.08}\right)^t}$

Brom: $B(0) = 10{,}000 \Rightarrow B(t) = 10{,}000\, b^t$

After 120 years: $B(120) = 30{,}000 = 10{,}000\, b^{120}$

$3 = b^{120}$

$b = \sqrt[120]{3}$

so $\boxed{B(t) = 10{,}000\left(\sqrt[120]{3}\right)^t}$

Brom has twice the pop. of Arcadia when: $\boxed{B(t) = 2A(t)}$

i.e.: $10{,}000\left(\sqrt[120]{3}\right)^t = 20{,}000\left(\sqrt[10]{1.08}\right)^t$

$\left(\sqrt[120]{3}\right)^t = 2\left(\sqrt[10]{1.08}\right)^t$

$\left(\dfrac{\sqrt[120]{3}}{\sqrt[10]{1.08}}\right)^t = 2$

$t \ln\left(\dfrac{\sqrt[120]{3}}{\sqrt[10]{1.08}}\right) = \ln 2$

$t = \dfrac{\ln 2}{\ln\left(\dfrac{\sqrt[120]{3}}{\sqrt[10]{1.08}}\right)} \approx 475.08 \Rightarrow$ $\boxed{\text{year } 2475}$