Corrections to
Introduction to Topological Manifolds (First edition)

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Changes or additions made in the past twelve months are dated.

Page 29, statement of Lemma 2.11: The second sentence should be replaced by "*If the open subsets of X are exactly those sets that satisfy the basis criterion with respect to $\mathcal{B}$, then $\mathcal{B}$ is a basis for the topology of X.*"

Page 29, paragraph before Exercise 2.15: Instead of "the topologies of Exercise 2.1," it should say "some of the topologies of Exercise 2.1."

Page 30, last sentence of the proof of Lemma 2.12: Replace U by $f^{-1}(U)$ (three times).

Page 30, first paragraph in the "Manifolds" section: Delete the sentence "Let X be a topological space."

Page 38, Problem 2-16(b): Replace part (b) by "Show that for any space Y, a map $f\colon X \to Y$ is continuous if and only if $p_n \to p$ in X implies $f(p_n) \to f(p)$ in Y."

Page 38, Problem 2-18: This problem should be moved to Chapter 3, because $\operatorname{Int} M$ and ∂M are to be interpreted as having the subspace topologies. Also, for this problem, you may use without proof the fact that $\operatorname{Int} M$ and ∂M are disjoint.

Page 40, last line of Example 3.1: Replace "subspace topology on B" by "subspace topology on C."

Page 46, second display: Replace k by $k/2$ (twice) and l by $l/2$ (twice). [The tangent and cotangent functions have period $\pi/2$, not π.]

Page 51, proof of Proposition 3.13, third line: $f_1(U_1), \dots, f_k(U_k)$ should be replaced by $f_1^{-1}(U_1), \dots, f_k^{-1}(U_k)$.

Page 51, proof of Proposition 3.14, last sentence: Replace "the preceding lemma" by "the preceding proposition."

Page 52, first paragraph after Exercise 3.8: In the first sentence, replace the words "surjective and continuous" by "surjective." Also, add the following sentence at the end of the paragraph: "It is immediate from the definition that every quotient map is continuous."

Page 52, last paragraph: Change the word "quotient" to "surjective" in the first sentence of the paragraph.

Page 53, line 1: Change the word "quotient" to "surjective" at the top of the page.

Page 53, Lemma 3.17: Add the following sentence at the end of the statement of the lemma: (*More precisely, if $U \subset X$ is a saturated open or closed set, then $\pi|_U\colon U \to \pi(U)$ is a quotient map.*)

Page 82, line 3 from bottom: Delete "$= \overline{U} \cap Z$" from the sentence beginning "Since $\overline{U \cap Z}$ . . . ."

Page 83, Example 4.30(a): In the first sentence, change "closed" to "open" and change $\overline{B}_\varepsilon(x)$ to $B_\varepsilon(x)$.

Page 85, statement of Corollary 4.34: "*countable collection*" should read "*countable union.*"

Page 99, Lemma 5.4: Replace part (d) by

(*d*) *For any topological space Y, a map $F\colon |\mathcal{K}| \to Y$ is continuous if and only if its restriction to $|\sigma|$ is continuous for each $\sigma \in \mathcal{K}$.*

Page 103, Proposition 5.11: In the statement of the proposition, change "*simplicial complex*" to "1-*dimensional simplicial complex.*"

Page 106, line 3 from bottom: Replace "even" by "odd."

Page 111, Figure 5.12: In $S(SK)$, the points inside the small triangles should be at the intersections of the three medians.

(1/9/25) **Page 112, second full paragraph:** Delete the sentence beginning with "Moreover."

(1/9/25) **Page 112, last paragraph, first sentence:** After "elementary subdivisions" add "or their inverses."

Page 114, Problem 5-2: Replace the statement of the problem by: "Let $\mathcal{K}$ be an abstract simplicial complex. For each vertex v of $\mathcal{K}$, let $\operatorname{St} v$ (the *open star* of v) be the union of the open simplices $\operatorname{Int}|\sigma|$ as σ ranges over all simplices that have v as a vertex; and define a function $t_v\colon |\mathcal{K}| \to \mathbb{R}$ by letting $t_v(x)$ be the coefficient of v in the formal linear combination representing x.

(a) Show that each function t_v is continuous.

(b) Show that $\operatorname{St} v$ is a neighborhood of v, and the collection of open stars of all the vertices is an open cover of $|\mathcal{K}|$."

Page 114, Problem 5-3: Delete the phrase "and locally path connected."

Page 120, Statement of Proposition 6.2(b): Replace $x \in \partial\mathbb{B}^2$ by $(x, y) \in \partial\mathbb{B}^2$.

Page 126, Proposition 6.6: Add the hypothesis that $n \geq 2$.

Page 131, Part 1 of the definition of the geometric realization: After "sides of length 1," insert "equal angles,".

Page 136, line 8 from bottom: Change the surface presentation in that line to $\langle S_1, S_2, a, b, c \mid W_1 c^{-1} b^{-1} a^{-1}, abcW_2 \rangle$.

Page 139, proof of the classification theorem: Replace the first sentence of the proof with "Let M be the compact surface determined by the given presentation."

Page 140, line 14: Change "Step 3" to "Step 2."

Page 149, Example 7.3: The first line should read "Define maps $f, g\colon \mathbb{R} \to \mathbb{R}^2$ by ...."

Page 155, line 3: Change $\Phi_g(f)$ to $\Phi_g[f]$.

Page 156, Figure 7.7: The labels $I \times I$, F, and X should all be in math italics.

Page 156, Exercise 7.2: Change the first sentence to "Let X be a path connected topological space."

Page 159, second line from bottom: "induced homeomorphism" should read "induced homomorphism."

Page 160, Proposition 7.18: In the statement and proof of the proposition, change $(\iota_A)*$ to $(\iota_A)_*$ three times (the asterisk should be a subscript).

Page 174, proof of Lemma 7.35: In the second-to-last line of the proof, change "Theorem 3.10" to "Theorem 3.11."

Page 176, Problem 7-5: Change "compact surface" to "connected compact surface."

Page 188, proof of Theorem 8.7: Replace the third sentence of the proof by "If $f\colon I \to \mathbb{S}^n$ is any loop based at a point in $U \cap V$, by the Lebesgue number lemma there is an integer m such that on each subinterval $[k/m, (k+1)/m]$, f takes its values either in U or in V. If $f(k/m) = N$ for some k, then the two subintervals $[(k-1)/m, k/m]$ and $[k/m, (k+1)/m]$ must be both be mapped into V. Thus, letting $0 = a_0 < \cdots < a_l = 1$ be the points of the form k/m for which $f(a_i) \neq N$, we obtain a sequence of curve segments $f|_{[a_{i-1},a_i]}$ whose images lie either in U or in V, and for which $f(a_i) \neq N$." Also, in the last line of the proof, replace "f is homotopic to a path" by "f is path homotopic to a loop."

Page 189, proof of Proposition 8.9: In the last sentence of the proof, change the domain of H to $I \times I$, and change the definition of H to

$$H(s,t) = (H_1(s,t), \ldots, H_n(s,t)).$$

Page 191, Problem 8-7: In the third line of the problem, change $\varphi(\gamma)$ to $\varphi_*(\gamma)$.

Page 192, line 4: Change the definition of φ to $\varphi(x) = (x - f(x))/|x - f(x)|$.

Page 199, second-to-last paragraph: In the second sentence, after "a product of elements of S," insert "or their inverses."

Page 208, Problem 9-4(b): Change the first phrase to "Show that $\operatorname{Ker} f_1 * f_2$ is equal to the normal closure of $\operatorname{Im} j_1 * j_2, \ldots$." Add the following hint: "[Hint: Let N denote the normal closure of $\operatorname{Im} j_1 * j_2$, so it suffices to show that $f_1 * f_2$ descends to an isomorphism from $(G_1 * G_2)/N$ to $H_1 * H_2$. Construct an inverse by showing that each composite map $G_j \hookrightarrow G_1 * G_2 \to (G_1 * G_2)/N$ passes to the quotient yielding a map $H_j \to (G_1 * G_2)/N$, and then invoking the characteristic property of the free product.]"

Page 213, proof of Proposition 10.5: In the second sentence of the proof, change $\{q\}$ to $\{*\}$.

Page 218, Figure 10.4: In the upper diagram, one of the arrows labeled a_i should be reversed.

Page 227, line 8: Replace $\overline{R} * \overline{S}$ by $\overline{R * S}$.

Page 233, last line: Change the last sentence to "This brings us to the next-to-last major subject in the book: ...."

Page 238, proof of Proposition 11.10, second line: Change "p maps ..." to "f maps ...."

Page 248, Example 11.26: Change $\mathcal{C}_\pi(\mathbb{P}^n)$ to $\mathcal{C}_\pi(\mathbb{S}^n)$.

Page 249, line 5: Change the formula to "$p(\varphi(\widetilde{q})) = p(\widetilde{q}) = q$" (not p).

Page 253, Problem 11-9: Change "path connected" to "locally path connected."

Page 265, Step 4: In the second line of Step 4, replace "as in Step 3" by "as in Step 2."

Page 268, proof of Theorem 12.11: The first and last paragraphs of this proof can be simplified considerably by using the result of Problem 3-15.

Page 272, first paragraph: The last sentence should read "It can be identified with a quotient of the group of matrices of the form $\left(\begin{smallmatrix}\alpha & \beta \\ \overline{\beta} & \overline{\alpha}\end{smallmatrix}\right)$ (identifying two matrices if they differ by a scalar multiple), and so is a topological group acting continuously on $\mathbb{B}^2$."

Page 284, just below the first displayed equation: Replace everything on that page below the first displayed equation with the following:

We have to show that p' is a covering map. Let $q_1 \in X$ be arbitrary, and let U be a neighborhood of q_1 that is evenly covered by p. We will show that U is also evenly covered by p'. Given a component $\widetilde{U}$ of $p^{-1}(U)$, let $U' = \pi(\widetilde{U}) \subset X'$; since π is an open map (Problem 3-15), U' is open in X'. Suppose $U'_1 = \pi(\widetilde{U}_1)$ and $U'_2 = \pi(\widetilde{U}_2)$ are any two such sets. If they have a point q' in common, then $q' = \pi(\widetilde{q}_1) = \pi(\widetilde{q}_2)$ for some $\widetilde{q}_1 \in \widetilde{U}_1$ and $\widetilde{q}_2 \in \widetilde{U}_2$. Since π identifies points of $\widetilde{X}$ if and only if they are in the same $\widetilde{H}$-orbit, there is some $\varphi \in \widetilde{H}$ such that $\widetilde{q}_2 = \varphi(\widetilde{q}_1)$. Then φ maps $\widetilde{U}_1$ homeomorphically onto $\widetilde{U}_2$, so $\pi(\widetilde{U}_2) = \pi \circ \varphi(\widetilde{U}_1) = \pi(\widetilde{U}_1)$. This shows that any such sets U'_1, U'_2 are either disjoint or equal. Since π is surjective, $p'^{-1}(U)$ is equal to the disjoint union of the sets $\pi(\widetilde{U})$ as $\widetilde{U}$ ranges over the components of $p^{-1}(U)$.

It remains only to show that for any such set $U' = \pi(\widetilde{U})$, $p'\colon U' \to U$ is a homeomorphism. The following diagram commutes:

$$\begin{tikzcd} \widetilde{U} \arrow[dd, "p"'] \arrow[dr, "\pi"] & \\ & U' \arrow[dl, "p'"] \\ U. & \end{tikzcd} \tag{12.8}$$

Since $p = p' \circ \pi$ is injective on $\widetilde{U}$, so is π; and $\pi\colon \widetilde{U} \to U'$ is surjective by definition. Because π is an open map, it follows that $\pi\colon \widetilde{U} \to U'$ is a homeomorphism. Since p and π are homeomorphisms in (12.8), so is p'.

Page 287, line 10: The sentence "Thus (i) corresponds to the rank 1 case" should read "Thus (ii) corresponds to the rank 1 case."

Page 289, Problem 12-5: Replace the statement of the problem by "Find a group Γ acting freely and properly on the plane such that $\mathbb{R}^2/\Gamma$ is homeomorphic to the Klein bottle."

Page 290, Problem 12-9: Replace the second sentence by "For any element $\widetilde{e}$ in the fiber over the identity element of G, show that $\widetilde{G}$ has a unique group structure such that $\widetilde{e}$ is the identity, $\widetilde{G}$ is a topological group, and the covering map $p\colon \widetilde{G} \to G$ is a homomorphism with discrete kernel."

Page 301, just above the third displayed equation: In the last sentence of the paragraph, replace $G_{i,p}\colon \Delta_p \to \Delta_p \times I$ by $G_{i,p}\colon \Delta_{p+1} \to \Delta_p \times I$.

Page 316, first paragraph: Change the fourth sentence to: "For $p > 0$, if $\alpha\colon \Delta_p \to \mathbb{R}^n$ is an affine p-simplex, set

$$s\alpha = \alpha(b_p) * s\partial\alpha$$

(where b_p is the barycenter of Δ_p), and extend linearly to affine chains."

Page 319, statement of Lemma 13.21: H^{n-1} should be H_{n-1}.

Page 320, first paragraph: In the last two lines, H^{n-1} should be H_{n-1} (twice).

Page 325, second to last displayed equation: Change $H_p(\mathcal{K}'')$ to $H_p^{\Delta}(\mathcal{K}'')$.

Page 330, paragraph after Exercise 13.4: Replace [Mun75] by [Mun84].

Page 332, line 1: The first word on the page should be "subgroups" instead of "spaces."

Page 333, line 7: Change "coboundary" to "cocycle."

Page 335, Problem 13-12: Add the hypothesis that $U \cup V = X$.

Page 344, Exercise A.7(a): Since this exercise requires the axiom of choice, it should be moved after exercise A.9.