

Getting Acquainted with The Geometer's Sketchpad 4: Chapter 1

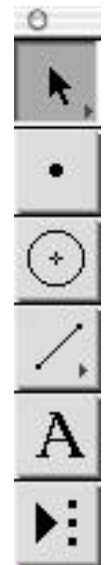
1.1 First Exploration with the Toolbox

Goal of this introduction

This is an introduction to Geometer's Sketchpad 4, the interactive geometry software from Key Curriculum Press. An important goal is to learn how the software works, but equally important is to get a feel for how such software opens up new possibilities for presenting and exploring geometry. To begin, we will look at the basic Sketchpad tools and use them to investigate some traditional topics from geometry.

When you open Sketchpad 4, you will see on the left a movable palette of geometry tools.

Choose a tool by clicking lightly on the tool icon with the mouse cursor. The square with the icon of the chosen tool changes to a darker shade (as the Arrow Tool is highlighted in the figure). In the figure, the arrow on the top is highlighted. Click in the toolbox a few times to see how the tools are highlighted. In some cases, if you press harder, a menu will pop out to the side. Just ignore these menus for now, they will be explained later.



We will begin learning how to use these tools by making a big mess of points, circles and segments. No serious drawing yet.



Point Tool. Let's begin with the point tool. Select the point tool by clicking once in the toolbox. Next, move the mouse (without pressing the mouse button) so that the cursor moves into the blank drawing region. Then click the mouse wherever you would like to create a point. Continue clicking to produce several more points for practice.



Circle Tool. Select the tool by clicking in the toolbox. You use this tool by *dragging*. Move the mouse cursor into the drawing window. Hold down the mouse button and move the mouse for a short distance and release the button. This draws a circle; notice that **it also draws two points at the beginning and end of the drag. The circle drawn has the first point as center and passes through the second point.** These special points determine the size and location of the circle. For practice, draw a number of circles of various sizes.



Segment Tool. This tool also works by dragging. Select the tool by clicking in the toolbox and then drag as you did with the circle to draw a line segment. Again the tool draws two points that are the end points of the segment Practice drawing some segments.



Arrow Tool for Selecting and Translating. This important tool does not draw, but *it moves objects that are already drawn.*

Dragging points. Select the tool and then move the mouse cursor until the tip of the arrow is at one of the points that you have drawn. Now press down the mouse button and **drag** to move the point. Try this with a point on a circle, the center of a circle, or an endpoint of a line segment. See how the circle or segment changes to follow the point as it moves.

Dragging a segment or circle as a whole object. Using the same tool, move the cursor to a location near the middle of a segment, press down the mouse button and drag. Observe how the segment moves. What changes and what stays the same? How does this differ from dragging the endpoint of a segment? Now drag a circle the same way by moving the cursor to a location on the circle not too close to the special point on the circle that determines the radius. How does the circle move?

Selecting and Moving Multiple Objects. You may well have done this already by accident. Using the Arrow Tool, if you click on an object, a point or a segment or a circle, the object is **selected**. The object changes appearance when it is selected. **To unselect all objects, click somewhere in the white space in the window.** Try selecting a point, a segment and a circle to see what this visual highlighting looks like. If an object or objects are already selected, and you click on another object, then this additional object is also selected while the first ones stay selected also. You can select as many objects as you like. Now while multiple objects are selected, if you take the mouse cursor, place it over one of these objects and drag, **all the objects will move in unison**. This is a useful feature when you want to use it, but sometimes one is surprised to see extra objects move.

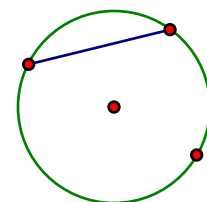
To be sure you are only selecting one object, first **click in the white space** to unselect all. Then you will select what you intend to select without surprises.

Use Tools with Existing Points

So far, our points, segments and circles have been created independently, but this need not be the case. Here are some examples of ways to create objects related to existing objects. In each case, **after making the construction, choose the selection arrow tool and drag your figure** in as many ways as you can to see how the objects are related.

In your figure you have a number of single points as well as the points creating circles and segments.

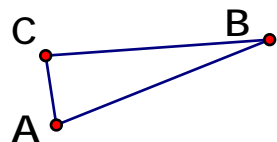
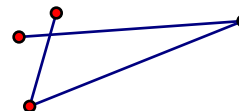
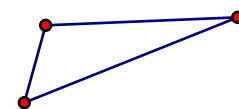
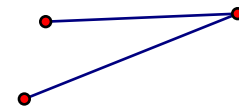
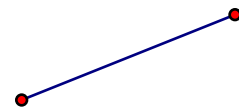
- Pick out **two single points**. Choose the **circle tool** and move the mouse cursor over the first point; when you are exactly over the point, the point will change color. Then press down the mouse over the first point and drag to the second point, which will also change color; then release. This should draw a circle with the first point as the center and the second point on the circle. There should be no new points in the sketch.
- Pick out a **segment**. Again use the **circle tool** to draw a circle with center at one endpoint and passing through the second endpoint. Now the segment is a radius of the circle. If you drag one of the points, both the segment and the circle move to keep this relationship.
- Pick out a **segment**. Choose the **segment tool**. Move the mouse cursor to a location on the segment (but not at an end point); when you are over the segment, it will change color. Click and hold the mouse button down. Drag the segment tool over a location in the “white space”; then release the mouse button. The new segment has two endpoints. The second endpoint is free to move anywhere, but the first endpoint is can only move along the original segment. This is an example of a **Point on Object**. Choose the arrow tool and move this point to see how this works. Also move the first segment and see how the point on object stays on the segment.
- Pick out a **circle**. Use the **segment tool** to create a new segment whose endpoints are points on the circle and thus both constrained to move on the circle. The circle will change color when the segment tool is over a location on the circle. If it does not change color, you will probably create a new point close to the circle but not on it.



Draw a Triangle

Create a new blank drawing window by choosing New Sketch in the File menu. Choose the **segment tool**.

- Draw a segment. This segment will have two endpoints.
- Draw a second segment starting at one of these endpoints. Move the mouse cursor over one of the endpoints of the first segment until the point changes color, press down, and drag to a new point in the white space of the window. You should now have a figure with 3 points and two segments with a common endpoint. (If you missed and have 4 endpoints and no common endpoint, choose Undo in the Edit menu and try again.)
- Draw a third segment connecting two existing endpoints to form a triangle.
- **Problems?** Test your construction using the **Drag Test**. Choose the Arrow Tool and one by one, drag each of the three points. The triangle should deform but not come apart if you do this. If there is a problem with an extra point and a disconnected triangle, use Undo repeatedly and try again.
- Now that we have a triangle, let's label the vertices ABC using the Text Tool.



Text Tool. *To label an object, click the object with the text tool.* All objects can have labels, but labels are hidden unless you make them visible. Choose the text tool and then move the mouse cursor over a point. Notice that the cursor is a hand with a pointing finger. As the tip of the finger moves over a point, the hand becomes dark. When the hand is dark, click and the label of the point will appear. Click on the **object** again (not the label) and the label will disappear. You can do the same with a segment or a circle. Notice that points are labeled with capital letters A, B, C, segments are labeled with lower-case letters j, k, l, and circles are numbered c1, etc.

To change a label, double-click the label with the text tool (do not double-click the object) and a "dialog box" will appear with a place to type a new label. You can change the label to another letter or to something longer and more descriptive, like *Interesting Point*. You can also click on Style and then get other choices to change the font, font size, style, etc. You can also add a subscript.

If the selection arrow is chosen, it automatically turns into the text tool when it passes over a label; then you can double-click to change the label. Notice that you can move the label somewhat by dragging the label with the text tool.

Editing or cleaning up your mess

The first sketch you made above is a messy figure on your screen. What can one do to clean up a mess, to change a figure or to correct mistakes?

- **New Sketch**

We have already used one simple option for obtaining a clean place to draw when we created the triangle ABC. We abandoned the first sketch and started a new one. We still have the choice of saving the first sketch or simply closing it without saving it. Or we can just ignore it and have multiple sketch windows open at the same time. Note: New Sketch creates a new Sketchpad document. It is also possible to create a new page in the same document using Document Options. This will be explained later (or you can find out how at any time using Sketchpad Help).

- **Undo**

If you choose **Undo** from the **Edit Menu**, your most recent action will be undone. If you keep choosing Undo, **even more of the construction will be undone, all the way back to a blank window** if you do it enough. If you feel you have undone too far, **Redo** (also in the Edit menu) will undo the undo.

- **Selecting and deleting objects**

You can delete an object with the **Delete key** or by choosing **Cut** from the Edit menu, but to delete an object this way, you have to select it first with the Arrow Tool. You can **select multiple objects** by clicking on them in turn, but you can also select multiple objects by **dragging a rectangle** with the selection arrow. The objects inside the rectangle will be selected.

To really get rid of everything, choose **Select All** from the **Edit Menu** and then press the Delete Key. (Note: **If you don't see Select All**, be sure that you have chosen the Selection Arrow tool before you drag down the menu. More about this later.) But is everything really gone? Try Undo and see what happens.

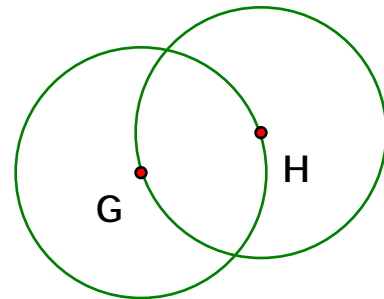
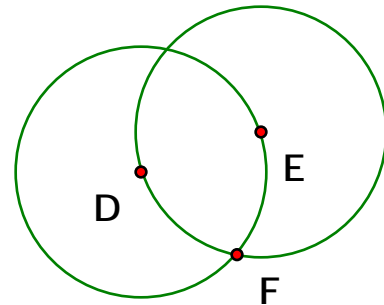
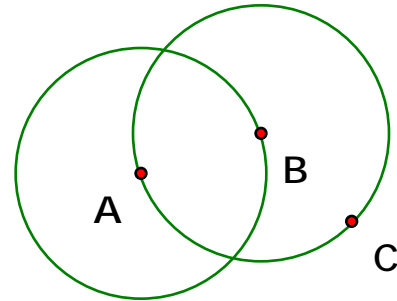
- **Hiding Objects**

Instead of deleting multiple objects, you can hide selected objects without removing them by choosing Hide from the Display menu. Hiding will be used frequently when we begin with geometrical constructions below.

1.2 Linked circles and equilateral triangles

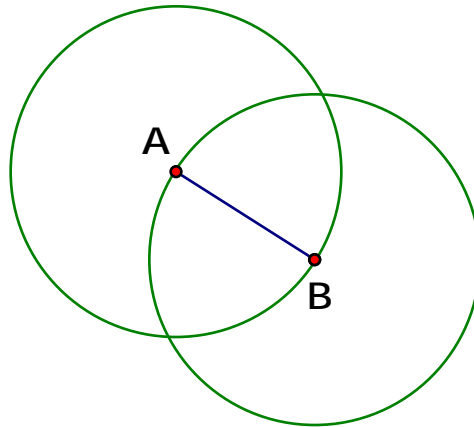
The freehand tools are electronic versions of Euclid's tools. Let's explore three kinds of linked circles and see how this leads to the classic construction of an equilateral triangle.

- Start a New Sketch.
- Construct a circle with center A through B and then drag a second circle from center B to a point C, outside the original circle, but so that the second circle passes through A. Now choose the Arrow Tool and drag each of the 3 points in turn. Does the second circle continue to pass through A as the points move? Do the circles remain the same size?
- Construct a circle with center D through E and then drag a second circle from center E to a point F on the original circle so that the second circle passes through D. Now choose the Arrow Tool and drag each of the 3 points. Does the second circle continue to pass through A as the points move? Do the circles remain the same size?
- Construct a circle with center G through H and then drag a second circle from center H to the point G. This figure only has two points defining the two circles. Now choose the Arrow Tool and drag each of the 2 points. Does the second circle continue to pass through A as the points move? Do the circles remain the same size?



Did the circles behave the way you expected? Upon reflection, do you see why they behave the way they do? If you think literally then if you move one point but not the others, then only the circle(s) that are constructed from that point will move. What appears on the screen may not reflect the logical connections. It is most important to think through why the circles in the last case always stay the same size, and why they do not in the other cases.

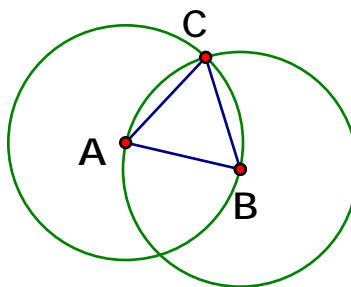
Construction of an equilateral triangle



- Start a New Sketch and draw a circle with center A through B and then construct a second circle with center B through A. Use the segment tool to construct the segment AB. This is like the last example from the previous exercise.

Important. Your sketch should only have *two* points at this stage, as in the third case of the previous exercise. If there are 3 points, you have constructed either the first or the second case above, and you should redo the figure.

- Now use the point tool and move the cursor over one of the points of intersection of the two circles. When you have arrived exactly at the intersection, the two circles will become highlighted. When this happens click the mouse button and a point will be constructed at the intersection.
- Now use the Text Tool to label this new point C and use the segment tool to construct the segments BC and CA.



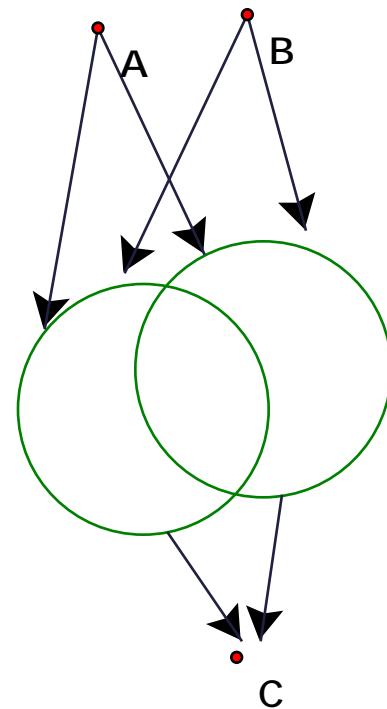
- If the triangle is constructed correctly, now matter what point you drag, the triangle will remain equilateral. The size of the triangle may change, but the 3 sides will remain equal and the 3 angles will remain equal. This is an example of a **Dynamic Geometry Figure**.
- Try dragging around the triangle to see what happens. Notice that the result of dragging A or B is different from dragging C. (For an explanation, see the next paragraph.) If your triangle always remains equilateral, it passes the **Drag Test**.
- **Problems?** If your figure does not result in an equilateral triangle that passes the Drag Test, then probably you clicked in a point at a location that was not quite on top of the existing point or intersection so that you have an extra point. There are some advanced methods for correcting errors in figures, but at this stage, the best course is to use Undo until a correct figure is reached. Then carry out the rest of the construction again.

Looking at Properties -- how a Dynamic Figure knows how to behave

In the equilateral triangle construction, did you notice that dragging C did not change the size of the triangle but dragging A and B did? We can see how Sketchpad decides how to move the figure by looking at the Properties of the objects in the figure.

- Choose the Text Tool and double-click on the point A. A dialog box will come up with two tabs at the top. The Label tab is selected, but click on the Object Tab to learn more about point A.
- In the Object tab window, it says that A is an independent object. It has no parents, but it has 4 children. What are the children?
- Let's compare this with point C. Double click on C and choose the Object tab for C. This window says that Intersection C is the intersection of Circle #1 and Circle #1. It lists those two circles as parents and the two segments with endpoint C as children.
- Here is a simpler example. Start by drawing a circle AB in a blank window; you actually draw two free (independent) points A and B and then draw the circle using A and B to determine the circle. This means that the circle is a child of the two points and that the parents of the circle are A and B. If a point is independent, like A or B, we can move it freely. If we drag A, the other free point B does not move, but the children of A (and their children) do move, even if they are joint children with B.
- But if we drag an object like the circle, we have to bring the parents A and B along, so the circle moves but does not change size.
- Now draw a second circle with center B through A, and let C be one of the intersection points of the circles. Since the parents of point C are the two circles, when we drag C, we also drag the two circles and consequently also drag A and B.
- The diagram on the right shows the parental relationships in this figure.
- A and B are the parents of each of the circles.
- The two circles are parents of C. Thus A and B are the grandparents of C.
- To learn more, just click on the "?" symbol in the Object window.
- There are other ways to find Properties in addition to the Text Tool method we have explained here, When an object is selected, the Properties item in the Edit menu will also produce the same properties dialog box.

Parents and Children

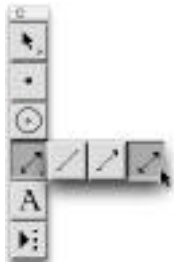


A and B are the parents of each of the circles and the two circles are the parents of C.

1.3 Drawing a line to construct a perpendicular bisector

We continue with the figure for the equilateral triangle that we constructed above.

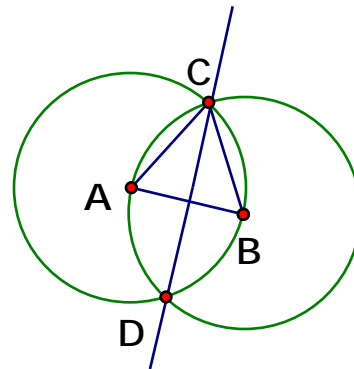
- Use the Point tool to construct a point by **clicking at the other point of intersection** of the two circles. Label this new point **D**.
- Draw the **line CD** (not the line *segment* CD) with the **Line tool**. **The Line tool is hidden behind the Segment tool**; here is how to find it.



Move the mouse cursor over the segment tool, but this time press the mouse button down and hold it. A menu pops out sideways! The menu is a row of 3 icons: one represents a segment, the second a ray (one arrowhead), and the third a line (two arrowheads). Drag the mouse to the line icon, and then release the mouse button. The toolbox will now show the line icon instead of the segment icon, so the **Line tool** is chosen, and when you drag with this tool you will draw lines instead of segments.

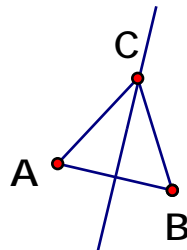
Likewise, if you drag out this menu of icons again but this time choose the **Ray tool**, you can draw rays by dragging. These tools collectively are called the **Straight Object tool**.

- Once you have chosen the line tool, **draw the line CD**.
- What is the angle between this line and the segment AB?
- What is this line called?



Hiding construction objects

- **Tidying up your figure.** Suppose you are mostly interested in the triangle ABC and the line CD and want to get rid of the "construction marks." You can **hide the circles**. Choose the Arrow Tool and click on the two circles to select them. Then choose **Hide Circles** from the **Display** menu. Hide point D in the same way. Now we only see the triangle and the line.



Do not DELETE the circles or the triangle will cease to be equilateral BECAUSE THE LOGICAL RELATIONS WILL BE DESTROYED. Hiding and deleting are different!

- To see the hidden parts of the figure again, choose **Display > Show All Hidden**.

Text on screen

- **Captions and titles.** To finish off the figure, we can add a title, such as Equilateral Triangle. We do this with the **Text tool**. Choose this tool and then, instead of clicking on an object, drag the mouse cursor diagonally down from left to right. A rectangle becomes visible. You can type text into this rectangle; you also edit your text at any time using the Text Tool. (Another way to get a new text box is to double-click at some empty location).
- When you are typing into the text box, a floating text style palette will appear. You can use this to change font, size, style and color and also can insert math symbols.
- You can change the size and shape of the text rectangle, and also move it around, using the Selection Arrow.
- You also can change the type size, font, etc. using items on the display menu.

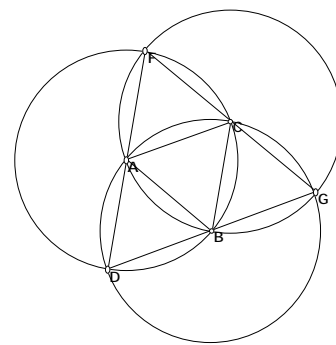
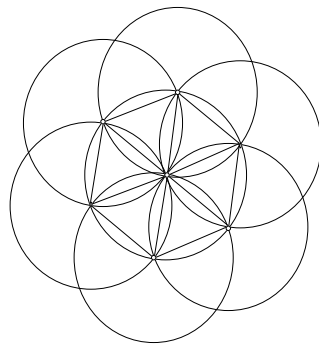
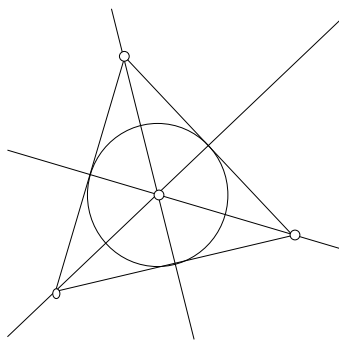
Saving your sketch

Save your sketch by choosing Save from the File menu and typing in a name in the indicated text box just as you would in a word processor or other program. Before clicking the Save button, first choose the disk and directory where you want the file to be stored.

- The **file extension** .gsp will be added automatically at the end of your filename by recent versions of Windows and by Macintosh OS X. For earlier operating systems, you may have to add the .gsp at the end by typing it in. Happily the files (if the names end in .gsp) should be usable across platforms, so you can use Windows sketches in Macs and vice-versa. Sketchpad sketches from Sketchpad version 3 can be opened in version 4, but Sketchpad 4 sketches cannot be opened using Sketchpad version 3.

Some figures to explore

More ideas. In the next section we will learn to do more complex constructions using menu commands, but the tools in the toolbox are very powerful, for they are an electronic version of the tools that Euclid knew. Here are some figures related to our triangle construction that suggest some ideas for further exploration with the toolbox.



1.4 Exploring the Construct menu

While we have begun with the tool bar, most of your Sketchpad constructions will probably be done by a different method—by using the Construct menu. Look at the Construct menu now by moving your mouse to the top of the screen and pressing down and holding the mouse button on Construct. Observe the names of the menu items. These are resources for constructing figures. The items may be dimmed at the moment because there is nothing in the sketch yet to construct with.

We begin with an example after a quick description of the general method.

General description

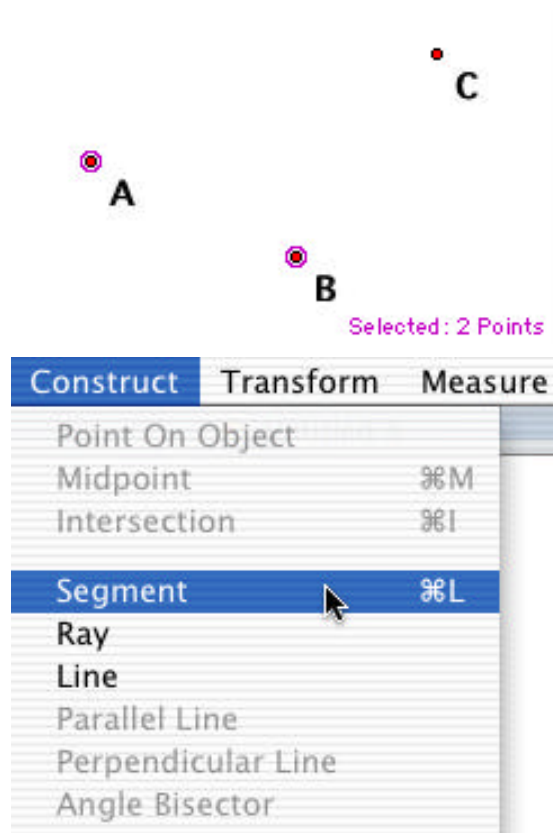
Step 1. Figure out which objects need to be selected in order to carry out the construction.

Step 2. Select some objects by clicking with the Arrow Tool. Be sure not to include “leftover” selections. It is often helpful to begin by *un-selecting everything by clicking on white space*. Then make your new selections.

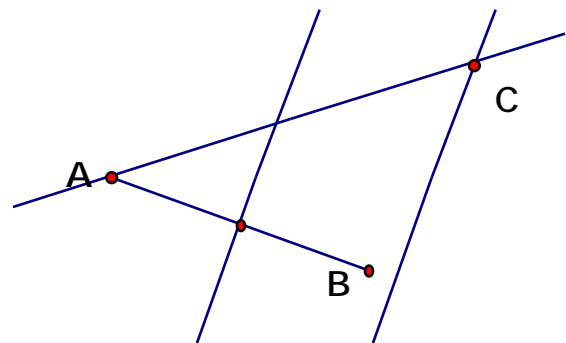
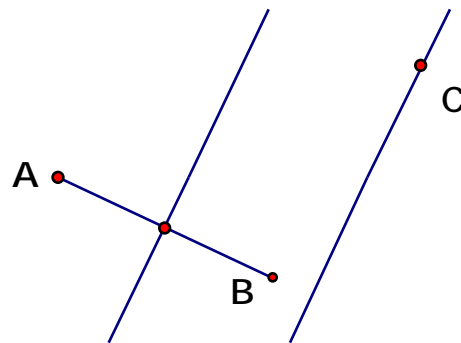
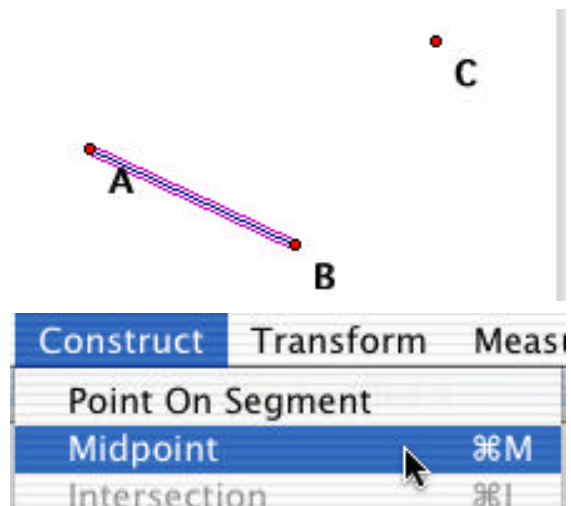
Step 3. When the objects are selected, choose an item from the menu to perform the task.

Let's practice.

- Create three points with the *Point tool*. Then *select two of the three points*. Notice how the selected points are highlighted with small circles. This is how you can tell whether the points are selected. Also, in the corner of the window is the text: Selected: 2 Points
- Next, choose Segment in the Construct menu. This will draw segment AB. We could have made the same segment the Segment tool; this is another way to do the same construction.



- When the segment is created, it is selected automatically. See the "handles" on the segment when it is selected. Practice clicking on "white space" to unselect everything and then click on the segment to select it again.
- If the segment alone is selected, there are two options available on the Construct menu: Point on Segment and Midpoint. Choose Midpoint.
- To construct the line through C perpendicular to AB, select the segment and point C. One option available on the Construct menu is **Perpendicular Line**. Choose this. In the same way you can construct the line through the midpoint perpendicular to AB; this constructs the **perpendicular bisector of AB**.
- Why do you think the menu items Segment and Point at Midpoint were *not* available with this selection of a point along with a segment?
- In the Construct menu are commands Segment, Ray and Line. For example, select points A and C and choose Line to construct the line AC instead of the segment AC.

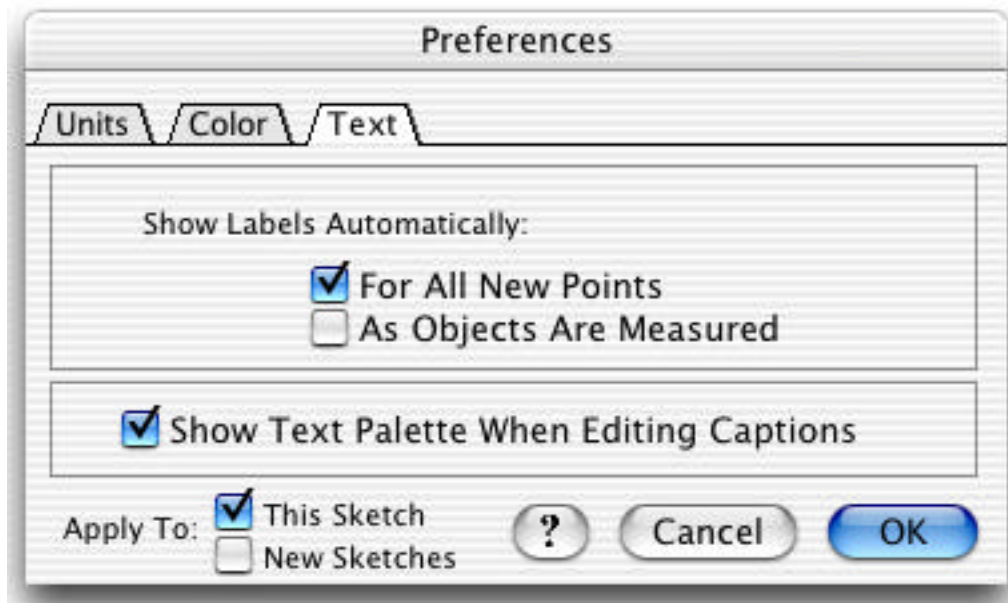


Getting help. Spend a little time exploring some of the other items on the construct menu. If an item is grayed out, this means that you have not selected what is needed for that item. To find out what to choose, read the information about the Construct menu in Sketchpad Help (from the Help menu).

1.5 Exploring midpoints and triangles

Triangle Area

Begin with a new sketch. Let's set Sketchpad so that points are labeled automatically. Choose Preferences from the Display menu. You will see something like this:



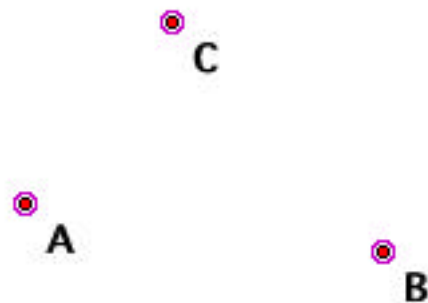
Check the box to Show Labels Automatically For All New Points. You can uncheck the box As Objects are Measured.

- As you can see, there are other interesting options here; you can return and explore them later.

Now we begin with a triangle. You know how to do this already with the segment tool, but we will show a shortcut with menus.

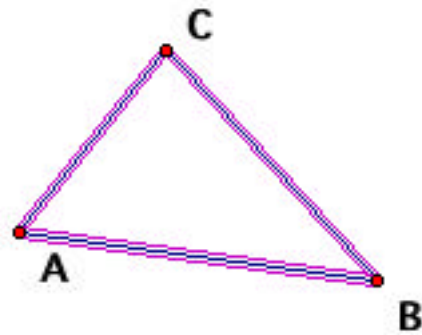
- Select the *Point* tool; now move the mouse cursor into the sketch.
- Hold down the Shift key as you click the mouse in three different places on the screen.

On the screen we see three points that are in bold outline, since three points are selected.



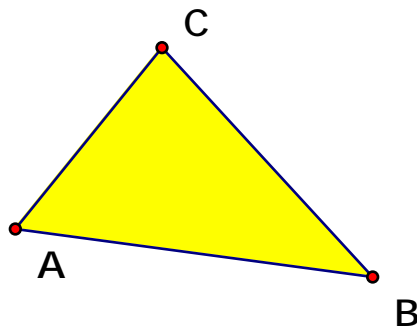
- Now choose **Segments** from the **Construct** menu.

The 3 segments are constructed at once. Notice the highlighting that indicates all the segments are selected.

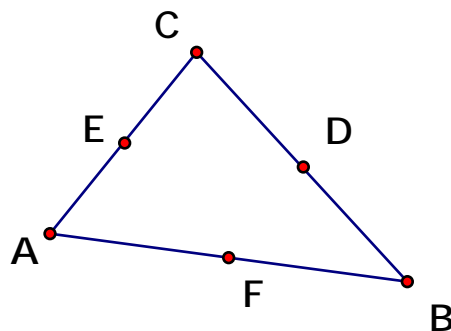


- Unselect the points by clicking in the white space.
- Select the three vertices of the triangle by clicking them. Choose **Triangle Interior** from the **Construct** menu. A flashing shaded region should appear. The flashing means that the interior is selected.
- Choose **Area** from the **Measure** menu. The **units** will be in cm or inches depending on your Preference choices. Notice that the area measurement requires the selection of the interior of the polygon (and the interior alone); the measurement is not available if the vertices and/or edges are selected.

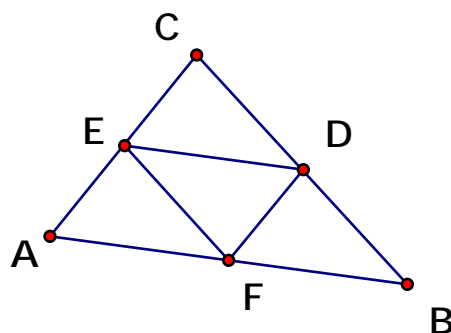
$$\text{Area}\triangle ABC = 6.29 \text{ cm}^2$$



- By clicking on white space, you can unselect the polygon interior so that it stops flashing. Click on the interior again and it again flashes. With the region still flashing, choose **Color** from the **Display** menu and see how the color of the region can be changed.
- Now choose **Hide Triangle** from the display menu. Notice that the area measurement is still on the screen even though the interior is hidden. If the interior is deleted, the area measurement will vanish.
- We want to construct the **midpoints** of sides of the triangle. This can be done with the **Midpoint** item in the **Construct** menu. To do this, **select all three segments but none of the three points**. Choose **Midpoints** from the **Construct** menu and see the three midpoints appear.



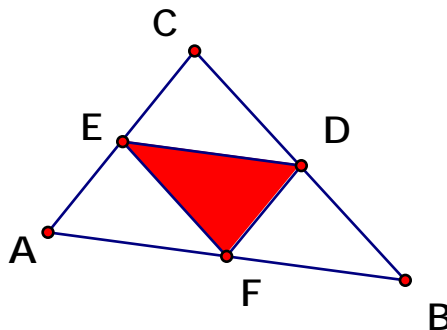
- Construct **the segments connecting the midpoints**. When the midpoints are created, they are selected. If they have become unselected, select the three midpoints again. Now choose **Segments** from the **Construct** menu. The three segments will be drawn all at once.



- Finally, construct the interior of the triangle DEF as you did before with ABC. Measure the area again; but this time, color the region a different color.

$$\text{Area } \triangle ABC = 6.29 \text{ cm}^2$$

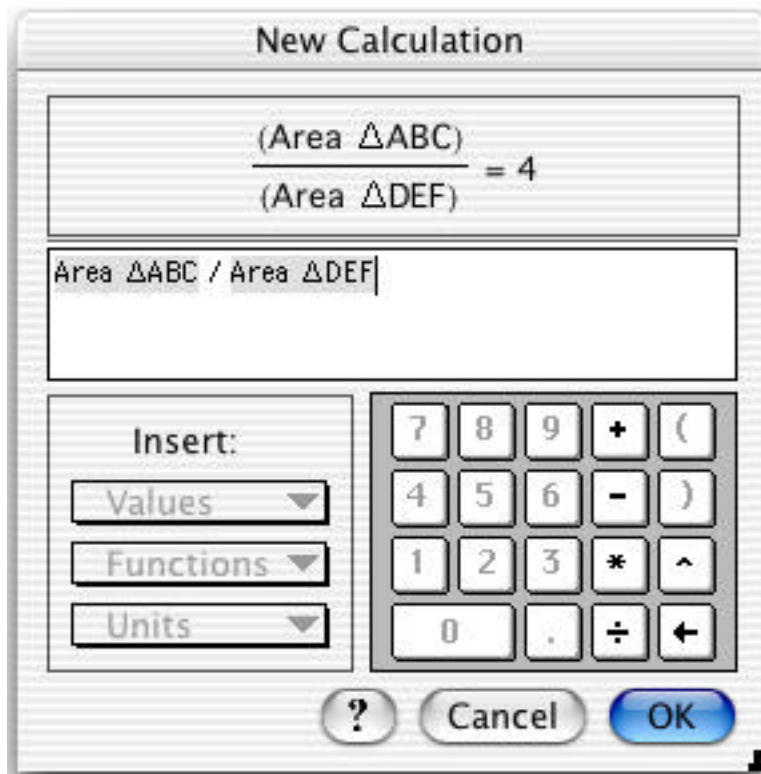
$$\text{Area } \triangle DEF = 1.57 \text{ cm}^2$$



- Drag some points around to see how the measurements change.

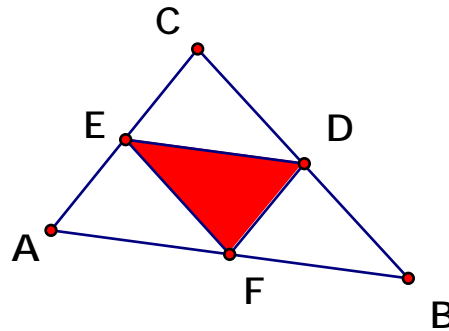
Using the Calculator

- What is the ratio of the areas of the two triangles? You may well see the answer already, but let's check by doing a calculation with Sketchpad. Choose **Calculate** from the **Measure** menu.
- A calculator pops up. Enter the area of ABC by clicking on the measurement on the screen. Be sure to click on the text object with the measurement, not the triangle itself.
- Next, click on the key in the calculator with the division sign (or just type in a / from the keyboard).
- Finish the calculation by clicking on the measurement of area DEF. Then press OK. (An alternate method to enter the measurements is to select the measurements and then select them from the Values menu in the calculator.)



- See that the ratio appears on the screen. If the location of the text is inconvenient, you can drag this calculated value with the Arrow Tool to any location you like.

$$\begin{array}{l} \text{Area } \triangle ABC = 6.29 \text{ cm}^2 \\ \text{Area } \triangle DEF = 1.57 \text{ cm}^2 \end{array} \quad \frac{(\text{Area } \triangle ABC)}{(\text{Area } \triangle DEF)} = 4.00$$

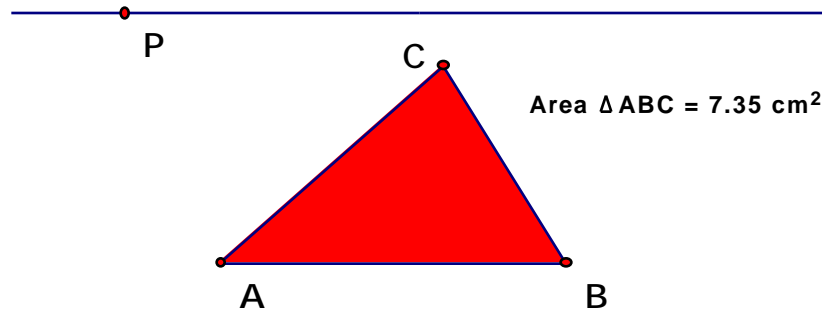


- Now drag A and/or B and/or C around with the mouse and see how the areas change but the ratio of areas does not.
- **Measuring lengths.** We can make other measurements and compare them. For example, we can compute the length of segment DE by selecting the segment and choosing **Length** from the **Measure** menu. We can measure the distance AF by selecting the points A and F and choosing **Distance** from the **Measure** Menu. How do these distances compare?
- Measure the perimeter of a polygon by selecting a polygon interior and then choosing **Perimeter** from the **Measure** menu. What is the ratio of the perimeters in the example above?
- **Measuring angles.** Measure angle BAC by selecting these three points (in order) and choosing **Angle** from the **Measure** menu. Measure some other angles. Do some angles remain equal even when the shape of the triangle is changed?
- **Explaining the area ratio with Sketchpad.** One can explain the area ratio by dividing triangle ABC into 4 smaller triangles with equal area. What are some of the ways to see experimentally with Sketchpad that the 4 areas are the same?

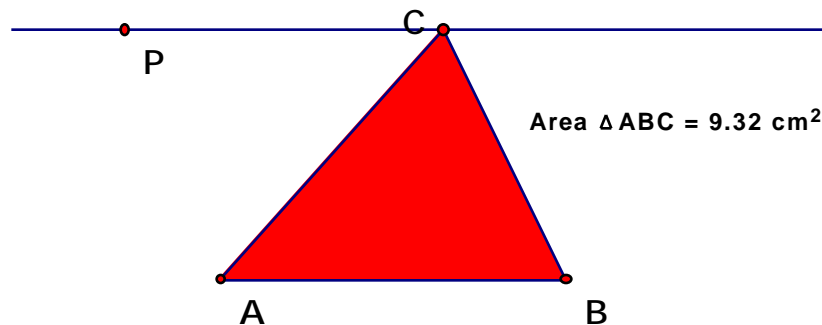
1.6 Shearing triangles

Now that we have learned how to measure area, let's look at a way that Sketchpad can help visualize triangles with equal base and altitude.

- Begin with a new sketch and construct a triangle ABC. Construct the interior of triangle ABC and measure the area of ABC as in the previous exploration. Notice that when you drag point C, the area will change.
- Then draw a point P in the plane and construct the line through P parallel to segment AB. (Do this by selecting P and the segment AB; then choose Parallel Line from the Construct menu.)



- Now merge point C to the parallel line by selecting C and the line and choosing Edit> Merge Point to Parallel Line.



- Now C is a point on the parallel line. When you drag C, the point moves along the line but the line itself does not move. Observe what happens to the area when C is dragged along the line. Can you explain this?
- If you drag point P, the area will change, but then all the positions of C with this new location for D will give the same area.
- Also measure the **perimeter** of the triangle. What is true about the triangle ABC when C is moved along the line so that the perimeter is a minimum?
- A deformation of a triangle by sliding a vertex parallel to opposite side is called **shearing**. This is an illustration of shearing and the effect of shearing on the area and perimeter of the triangle.
- **Further explorations:** We can add features to the sketch to make it more visually compelling. For example, in the next section, it we see how to add an altitude and measure base and altitude.
- **Shear a parallelogram.** Construct a parallelogram ABCD in such a way that dragging vertex C shears the parallelogram parallel to AB and does not change the area.

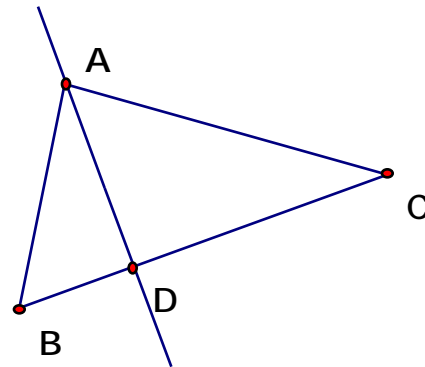
1.7 Areas and altitudes

Let's relate the area computations for triangles to the usual formula with base and altitude.

- Create a triangle ABC as before. Then select point A and segment BC and choose Perpendicular Line from the Construct menu to construct the altitude through A.

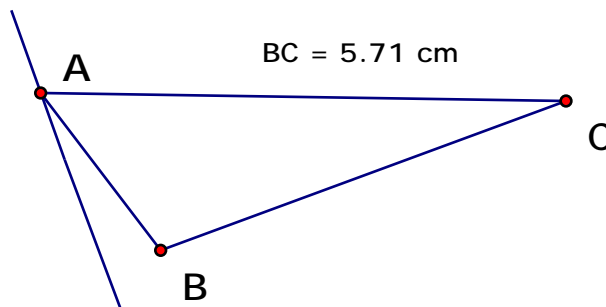
$$BC = 5.71 \text{ cm}$$

$$AD = 2.85 \text{ cm}$$

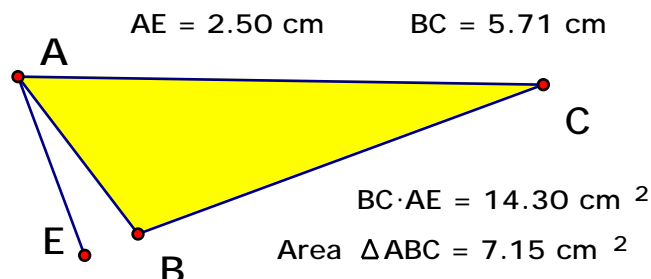


- Select the altitude line and the segment BC and choose Intersection from the Construct menu to form point D. Also measure the distance BC (the base) and the distance AD (the altitude).

- This measures the distances we need, but there is a problem. Drag A so that angle ABC becomes obtuse, then point D disappears and so does the measurement of distance AD! This is of course logical and correct. In this case there is no intersection of the perpendicular line and the segment BC. But even though correct, it is inconvenient, for it does not measure the altitude that we want.



- Let's redo the last steps. Choose Undo from the Edit menu several times until point D disappears. (The altitude line should remain.)
- Now select points B and C and construct the *line* BC (not the segment!) so that the line is on top of the existing segment.
- Then move point A as in the figure above so that the perpendicular line only intersects line BC but not segment BC. Then construct point E as the Intersection of the two lines.
- Now hide both lines and construct segment AE. This is an altitude that will work wherever you drag A, B and C. Measure distance AE and compare the product of the distances BC and AE with the area of triangle ABC.



1.8 Exploring Quadrilaterals

We have given a lot of step-by step instructions up to now. This section presents interesting mathematical situations to explore and investigate in a variety of ways. Find the answers to the questions and explain why they are true.

Midpoint quadrilaterals

Begin with a general quadrilateral ABCD. Let EFGH be the midpoints of the sides. The quadrilateral EFGH is called the **midpoint quadrilateral**.

- What kind of quadrilateral is EFGH?
- How does the area of EFGH compare to the area of ABCD?
- The diagonals of ABCD are the segments AC and BD. Can you find a relation between the lengths of the diagonals and the perimeter of EFGH?
- For what kind of ABCD is the midpoint quadrilateral a rectangle? A rhombus? A square?
- 3D quadrilaterals: Cut out a quadrilateral ABCD from paper and fold it on the diagonal AC so that the sides form a quadrilateral in space. What is still true about the midpoint quadrilateral? Can you construct another quadrilateral that can be taped to ABCD to form a tetrahedron? What are the relationships between the two space quadrilaterals if they form a tetrahedron?

Diagonals and quadrilaterals

Several theorems about the diagonals of special quadrilaterals are proved in geometry courses. Turn this upside down and construct quadrilaterals from their diagonals. In each case, you will need to construct two segments with some special properties to make the special quadrilaterals.

- Construct the diagonals of a parallelogram and then construct the parallelogram.
- Construct the diagonals of a kite and then construct the kite.
- Construct the diagonals of a rhombus and then construct the rhombus.
- Construct the diagonals of a rectangle and then construct the rectangle.
- Construct the diagonals of a square and then construct the square.

Construction of special quadrilaterals

Think up other constructions for special quadrilaterals:

- How many other ways can you find to construct a square, a rectangle, a rhombus, a kite, or a parallelogram?