# PROPOSED SYLLABUS FOR THE 1st YEAR GRADUATE ALGEBRA SEQUENCE

## 1. Topics

| | |
|---|---|
| **Group theory** | Sylow theorems. Solvable and nilpotent groups, normal and central series, free groups, simple groups, Jordan-Hölder theorem. Direct and semi-direct products, extensions. |
| **Category theory** | Categories and functors, natural transformations, universal properties, products and coproducts. |
| **Rings and modules** | ◦ Polynomial rings, elementary symmetric polynomials. Euclidean, Principal ideal, and Unique factorization domains, Gauss lemma. |
| | ◦ Structure theorem for modules over PID: elementary divisors and invariant factor forms. Noetherian and Artinian rings and modules, Hilbert basis theorem, simple modules, composition series, and Jordan-Hölder theorem for modules. |
| | ◦ Vector spaces and linear operators, characteristic and minimal polynomials, Cayley-Hamilton theorem, canonical Jordan form, Rational Canonical Form. |
| | ◦ Semi-simple rings, Artin-Wedderburn theorem. |
| **Field theory** | Field extensions: finite, separable, normal, algebraic and transcendental. Existence of algebraic closure. Galois theory. Finite fields. Hilbert theorem 90, linear independence of characters, Kummer and cyclotomic extensions. Solvability of equations in radicals. |
| **Group actions and representation theory** | Group algebras, irreducible representations, Schur′s lemma. Complex representations of finite groups: complete reducibility of representations (Maschke′s theorem), character theory. |
| **Commutative algebra: rings** | Local rings and Nakayama lemma, Integral extensions, Krull dimension, Noether normalization lemma, Hilbert Nullstellensatz, localization. Prime ideal spectrum and Zariski topology, Algebraic sets and rings of regular functions. Discrete valuation rings and Dedekind domains. |
| **and modules** | Tensor product, flatness, local properties of modules, exterior and symmetric powers. Graded rings and modules, Hilbert functions and polynomials. |
| **Homological algebra** | Exact sequences, 5-lemma and snake lemma, projective and injective modules, resolutions, chain complexes, (left and right) exact functors, adjoint functors, adjointenss of Hom and Tensor functors, Tor and Ext. |

## 2. References

General:

- M. Artin, *Algebra* (more of an undergraduate textbook)
- D. Dummit and R. Foote, *Abstract Algebra*
- T. Hungerford, *Algebra*
- S. Lang, *Algebra*
- J. Rotman, *The Theory of Groups, an introduction* (group theory)

Representation theory:

- J. Alperin and R. Bell, *Groups and Representations*
- R. Pierce, *Associative algebras* (nice treatment of Wedderburn's theory)
- J.-P, Serre, *Linear Representations of Finite Groups*

Commutative algebra:

- M. Atiyah, I. Macdonald, *Introduction to Commutative Algebra*
- D. Cox, J. Little, D. O'Shea, *Ideals, Varieties, and Algorithms*
- D. Eisenbud, *Commutative Algebra with a view towards Algebraic Geometry*
- M. Reid, *Undergraduate Algebraic Geometry*
- M. Reid, *Undergraduate Commutative Algebra*

Homological algebra:

- C. Weibel, *An introduction to homological algebra*