

# Math 136, Spring 2015, Homework 8

## For practice- from SHE

1. Section 15.1, problems 7, 19, 21, 25, 35, 36.
2. Section 15.2, problems 5, 9, 35, 41, 45, 51.
3. Section 15.3, problems 13, 27, 39, 41-46.
4. Section 15.4, problems 1, 5, 7, 17, 23, 46.
5. Section 15.5, problems 3, 9, 13.
6. Section 15.6, problems 3, 15, 23, 30, 33.
7. Problems 41 and 57 in Section 15.4 together with a picture for the function  $f(x, y) = 9 - x^2 - y^2$  with  $(x_0, y_0) = (1, 2)$ .

## To hand in

1. Let  $g : \mathbf{R}^2 \rightarrow \mathbf{R}$  be the function defined by

$$g(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0). \end{cases}$$

Show that

$$g_y(x, 0) = \lim_{h \rightarrow 0} \frac{g(x, h) - g(x, 0)}{h} = x$$

and similarly that  $g_x(0, y) = -y$ . Hence, show that  $g_{yx}(0, 0) = 1$  and  $g_{xy}(0, 0) = -1$ .

2. Problems 41 and 57 in Section 15.4 together with a picture (you can copy the one in the book).
3. Let  $f(x, y) = 2x + 3y + 23$  be the temperature on the floor of a room at the point  $(x, y)$ , where  $x$  and  $y$  are measured in meters (m), and  $T$  is measured in degrees Centigrade ( $^{\circ}C$ ), and you are standing at the point  $(1, 1)$ 
  - (a) You are going to walk 1 meter in any direction you choose. Which way do you walk to warm your feet the most and how much higher is the temperature there in comparison to where you were initially?
  - (b) Describe the set of points on the floor near you where the temperature is  $28^{\circ}C$
4. Suppose that  $T = f(x, y)$  represents the temperature on the floor of a room at the point  $(x, y)$ , where  $x$  and  $y$  are measured in meters (m), and  $T$  is measured in degrees Centigrade ( $^{\circ}C$ ). Assume that  $T$  is differentiable. You are given the following data:

$$f(0, 0) = 20^{\circ}C, \quad f_x(0, 0) = 3^{\circ}C/m \quad f_y(0, 0) = 4^{\circ}C/m.$$

You are located at the origin (the center of the floor). Using only the information given, answer the following questions as best you can.

- (a) If you are only allowed to move a maximum distance of 1 m (in any direction) along the floor, can you move to a place where the temperature is (approximately)  $25^{\circ}C$ ? Explain.
- (b) Describe the set of points on the floor near you where the temperature is (approximately)  $20^{\circ}C$ .

## Bonus

Suppose that  $f : \mathbf{R}^n \rightarrow \mathbf{R}$  is a continuous function and let  $I$  be an open interval of the form  $I = (a, b)$  where  $a < b$ . Using the definition of continuity, show that the preimage of  $I$

$$f^{-1}(I) = \{\mathbf{x} \in \mathbf{R}^n : f(\mathbf{x}) \in I\}$$

is an open set.