Math 136, Spring 2015, Homework 6

For practice, from Chapter 5

1. Problems 1.5, 1.6, 1.7.
2. Problems 2.1, 2.2, 2.3, 2.5.
3. Problems 3.1, 3.2, 3.3, 3.4.
4. Problems 4.1, 4.2, 4.3, 4.4.
5. Let $U \subset \mathbf{R}^4$ be the subspace spanned by the two column vectors

$$A_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } A_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} .$$

Let $P : \mathbf{R}^4 \to \mathbf{R}^4$ denote the linear map given by orthogonal projection onto U. Find the matrix of P with respect to the standard basis for $\mathbf{R}^4$.

To hand in

1. Problem 5.5.
2. Let $\langle\, ,\, \rangle$ be a positive definite inner product on the vector space V. Let $L : V \to V$ be a linear operator that satisfies the condition

$$(u, L(v)) = (L(u), v) \text{ for all } u, v, \in V .$$

(Such an operator is said to be *self-adjoint*.)

Let v_λ and v_μ be eigenvectors associated to the eigenvalues λ and μ of L, with $\lambda \neq \mu$. Show that $v_\lambda \perp v_\mu$.

3. Let V be the vector space of continuous functions on the closed interval $[-1, 1]$, with scalar product defined by

$$(f, g) = \int_{-1}^{1} f(x)g(x)\, dx .$$

(a) Apply the Gram-Schmidt orthogonalization process to the set $\{1, x, x^2, x^3\}$ to obtain an orthogonal set of four polynomials, $\{p_0(x), p_1(x), p_2(x), p_3(x)\}$.

(b) Verify that p_k is a solution of the differential equation

$$(1 - x^2)y'' - 2xy' + \lambda y = 0, \text{ with } \lambda = k(k + 1) .$$

Remark: After multiplication by constants the functions $p_k(x)$ are called *Legendre polynomials* and the differential equation is called *Legendre's equation.*