Math 136, Spring 2015, Homework 3

For practice, do all of the problems at the end of Sections 3-8 in Chapter 2 in Linear Algebra Done Wrong.

Computation wise, you should be able to take an m by n matrix A, reduce it to echelon form and find the spaces Null A = ker A, Im A=Ran A= the column space of A, the row space of A and their dimensions (Section 7). You should be able to use row reduction to solve a system of linear equations (Section 2), invert a matrix (Section 4) and decide if a set of vectors is linearly independent (Section 3). You should also be able to write a change of basis map and write a matrix of a transformation with a matrix other than the standard one (Section 8).

It is a good idea to have a library of transformations to think about when you are answering True/False questions or coming up with ideas for proofs. Rotations, reflections, stretching or shrinking spaces (all three from $\mathbf{R}^n$ to itself), projections (from $\mathbf{R}^n$ to a subspace) and "inserting" $\mathbf{R}^n$ into $\mathbf{R}^m$ with $n \leq m$ (example from Tuesday's lecture) are standard examples of linear transformations. In fact, any linear transformation from $\mathbf{R}^n$ to $\mathbf{R}^m$ is a composition of several of these, which we will see later. Also, have some examples of bases for the subspaces of $\mathbf{R}$, $\mathbf{R}^2$ and $\mathbf{R}^3$ to think about. The nice thing about these, of course, is that we can visualize them geometrically.

To hand in

1. Problems 7.4, 7.5 and 7.6 in Section 7.

2. (a) Show that the complex numbers $\mathbf{C}$ may be viewed as a 2-dimensional real vector space.

   (b) Let $L : \mathbf{C} \to M_{2\times 2}$ be the map defined by

   $$L(x+iy) = \begin{pmatrix} x & y \\ -y & x \end{pmatrix}$$

   Verify that L is a linear map. What is the rank of L?

   (c) Show that L satisfies the identity $L(z_1 z_2) = L(z_1)L(z_2)$ for all $z_1, z_2 \in \mathbf{C}$.

3. Let A be a $n \times n$ matrix and let A_i denote the i-th row of A. We say that A is an *orthogonal* matrix if it satisfies the identities

   $$A_i \cdot A_i^t = 1$$

   and

   $$A_i \cdot A_j^t = 0 \text{ for all } i \neq j.$$

   The set of all $n \times n$ orthogonal matrices is called the *orthogonal group* and is denoted by $O(n)$.

   (a) Show that $A^{-1} = A^t$ for all $A \in O(n)$.

   (b) Show that $A^t \in O(n)$ for all $A \in O(n)$.

   (c) Show that $AB \in O(n)$ for all $A, B \in O(n)$.