Math 136, Spring 2015, Homework 2

For practice, do all of the problems at the end of Sections 6 and 7 in Chapter 1 and Section 2 of Chapter 2 in Linear Algebra Done Wrong.

In particular, you should be comfortable with working with matrices (coming up with all the examples in Section 6 of Chapter 1 problems) and using the augmented matrix to solve a system of linear equations (Section 2 of Chapter 2).

To hand in

1. Problem 6.11 in Chapter 1. You may want to look at Section 8, where the matrix for rotating around the vector $(0\,0\,1)^T$ is given as an example. There is one hint in the book. Here is another one: Break up the vector you are rotating into two components: one parallel to $(1\,2\,3)^T$ and the other perpendicular. The parallel component stays put. Try to rotate a perpendicular vector around $(1\,2\,3)^T$.

2. Problem 2.1(c) in Chapter 2.

3. Given a linear transformmation $T : V \to W$, the kernel of V is $\ker T = \{\mathbf{v} \in V : T(v) = \mathbf{0}\}$ and the image of V is $\operatorname{Im} T = \{T(\mathbf{v}) : \mathbf{v} \in V\}$. The kernel is a subset of V and the image is a subset of W.

   (a) Show that the kernel and the image of a transformation are vector subspaces of the domain and target set, respectively.

   (b) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ defined by $(x\ y\ z)^T \to (x\ 0\ y)$. Describe the image and kernel of this transformation, both geometrically and using a basis for each, and find their dimensions.

   (c) Let W be the set of all functions from $\mathbf{R}$ to $\mathbf{R}$ with continuous derivatives. Let D be the differentiation map. Then, D is a linear transformation (Why?). What are the image and kernel of D?

Bonus Do 6.10 with an arbitrary vector instead of $(1\,2\,3)^T$.