Math 136, Spring 2015, Final Review

The in class part of the final exam will cover Chapter 15, Sections 16.1- 16.6 and Sections 17.1-17.4 from SHE together with center of mass from Section 17.5.

Topics: Functions of two and three variables, partial derivatives, chain rule, gradient, level curves and level surfaces, tangent planes to surfaces, using tangent planes for approximating functions, critical points, classifying critical points, optimization, double integrals in rectangular and polar coordinates, changing the order of integration as an integration technique, mass and center of mass.

The book has numerous practice problems. You can also look at old Math 126 final exams for standard questions on optimization and double integrals. Here are some more review questions:

1. Let $w = \left(x^2 + y^2 + y^2\right)^{n/2}$, where n is a positive integer. Evaluate

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\,.$$

   Simplify as much as possible.

2. Find the critical points of

$$f(x,y) = \sin x\,\sin y\,, \quad 0 < x < 2\pi, \quad 0 < y < 2\pi\,.$$

   Which, if any, of these points is a saddle point of $f(x,y)$?

3. Evaluate the double integral

$$\iint_\Omega \sqrt{xy}\,dxdy\,, \quad \Omega = \{(x,y)\,:\,0 \le y \le 1\,,\ y^2 \le x \le y\}\,.$$

4. Let T be the temperature in a room viewed as a function of (x,y,z), and suppose that

$$\nabla T(1,1,1) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\,.$$

   Suppose further that an insect is flying around the room and that its position $\mathbf{r}(t)$ and velocity $\mathbf{r}'(t)$ satisfy the conditions

$$\mathbf{r}(1) = \mathbf{i} + \mathbf{j} + \mathbf{k} \text{ and } \mathbf{r}'(1) = -2\mathbf{i} + 6\mathbf{j} + 7\mathbf{k}\,.$$

   A tiny thermometer recording the temperature of the room is attached to the insect. Find the rate of change at time $t = 1$ of the temperature recorded by the thermometer.

5. Let $w = w(x)$, where $x = x(s,t)$, and assume that these functions have continuous second partials. Show that

$$\frac{\partial^2 w}{\partial s \partial t} = \frac{d^2 w}{dx^2}\frac{\partial x}{\partial s}\frac{\partial x}{\partial t} + \frac{dw}{dx}\frac{\partial^2 x}{\partial s \partial t}\,.$$

6. Let T be the temperature in a room viewed as a function of (x,y,z), and suppose that

$$\nabla T(1,1,1) = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}\,.$$

   Suppose further that an insect is flying around the room and that its position $\mathbf{r}(t)$ and velocity $\mathbf{r}'(t)$ satisfy the conditions

$$\mathbf{r}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k} \text{ and } \mathbf{r}'(0) = \pi\mathbf{i} + e^2\mathbf{j} + \sqrt{2}\mathbf{k}\,.$$

   A tiny thermometer recording the temperature of the room is attached to the insect. Find the rate of change at time $t = 0$ of the temperature recorded by the thermometer. (*Time is measured in seconds and position in meters.*)

7. Find the volume of the region

$$\Omega = \{(x,y,z) : 0 \le z \le x^2 + y^2\,, \quad 0 \le y \le x^2\,, \quad 0 \le x \le 1\}\,.$$

8. A plate made of mass density $\rho(x, y) = \sqrt{x^2 + y^2}$ is in the shape of the region

$$\Omega = \{(x, y, z) : 0 \leq x, 0 \leq y, x^2 + y^2 \leq 1\}.$$

Find the total mass of the plate and its center of mass.
Note: We did this type of questions in Math 134 using single integrals. The formulas using double integrals are symmetric and easier to remember.

9. Let

$$f(x, y, z) = x^2 y z^3,$$

   (a) Find the equation of the tangent plane to the level surface $f(x, y, z) = 1$ orthogonally at the point $(1, 1, 1)$.

   (b) Find parametric equations for the line intersecting the level surface $f(x, y, z) = 1$ orthogonally at the point $(1, 1, 1)$

10. Let $w = f(x, y)$, $x = g(u, v)$, and set $k(u, v) = f(g(u, v), v)$. Assume that f and g have continuous first and second partial derivatives. Find expressions for the first and second partial derivatives $\frac{\partial}{\partial v} k(u, v)$ and $\frac{\partial^2 k(u, v)}{\partial v^2}$ in terms of partial derivatives of f and g.

11. Let $f(x, y, z) = x + y^2 + z^3$ and $g(x, y, z) = x^3 + y^2 + z$. The level surface $g(x, y, x) = 3$ intersects the level surface $f(x, y, z) = 3$ at the point $(1, 1, 1)$. What is the cosine of the angle between the normals to these two surfaces at the point $(1, 1, 1)$?

12. The function $f(x, y) = 2x^2 - 2x^3 + x^4 - 2x^2 y - 8y^2 + 8xy^2 - 3x^2 y^2 + 8y^3 - 4y^4$ has two critical points: one at $(0, 0)$ and one at $(1, 1)$. Which, if any, of these points is a local maximum for f?