Math 135, Winter 2015, Homework 6

For practice - do not hand in

From TP, Chapter 4

1. Page 210, problems 1, 9
2. Page 220, problems 6, 23, 32, 30, 34.
3. Page 231, problems 1, 20, 23, 25, 33.
4. Page 240, Problems 1, 7, 20.
5. Page 246, Problems 2, 13, 20

To hand in

1. Consider the initial value problem

$$x'' - 2x' - x = t\,, \quad x(0) = 0\,, \quad x'(0) = 0\,.$$

   This is equaivalent to the system of first order differential equations

$$\begin{cases} x' &= y \\ y' &= x + 2y + t \end{cases} \qquad \begin{cases} x(0) &= 0 \\ y(0) &= 0\,, \end{cases}$$

   whose solution is the limit of a sequence of Picard iterates, $x_n(t)$, $y_n(t)$, starting with the initial guess

$$\begin{cases} x_0(t) &= 0 \\ y_0(t) &= 0\,. \end{cases}$$

   (a) Give the formula expressing $x_{n+1}(t)$ and $y_{n+1}(t)$ in terms of $x_n(t)$ and $y_n(t)$.

   (b) Use the formula in (a) to find the first three Picard iterates $x_n(t)$ and $y_n(t)$, $n = 1, 2, 3$.

2. Suppose that y_1 and y_2 are a fundamental set of solutions of the ODE $y'' + p(t)y' + q(t)y = 0$ on the interval $-\infty < t < \infty$, where p and q are continuous for all t. Prove that there is one and only one zero of y_1 between two consecutive zeros of y_2.

   Hint: *Differentiate the function $f(t) = y_2(t)/y_1(t)$ and use Rolle's Theorem.*

3. Let y_1 and y_2 be solutions of $y'' + py' + qy = 0$, where p, q are continuous functions on the interval $I = (a, b)$. Show that if there is a point in I where both y_1 and y_2 both vanish or where both have maxima or minima, then one of y_1 and y_2 is a multiple of the other.

4. Find the general solution of $t^2y'' + 3ty' + y = 0$, $t > 0$.