Math 407A: Linear Optimization

Lecture 4: LP Standard Form [2]

[2] Author: James Burke, University of Washington

1. LPs in Standard Form
2. Minimization $\rightarrow$ maximization
3. Linear equations to linear inequalities
4. Lower and upper bounded variables
5. Interval variable bounds
6. Free variable
7. Two Step Process to Standard Form

LPs in Standard Form

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We say that an LP is in *standard form* if its matrix representation has the form

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max	$c^T x$	It must be a maximization problem.
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max	$c^T x$	It must be a maximization problem.
s.t.	$Ax \leq b$	Only inequalities of the correct direction.
	$0 \leq x$	All variables must be non-negative.

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Every LP can be Transformed to Standard Form

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- *minimization* $\rightarrow$ *maximization*

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- *minimization → maximization*

  To transform a minimization problem to a maximization problem multiply the objective function by -1.

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- *minimization → maximization*

  To transform a minimization problem to a maximization problem multiply the objective function by -1.

- *linear inequalities*

  If an LP has an inequality constraint of the form

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \geq b_i,$$

  it can be transformed to one in standard form by multiplying the inequality through by -1 to get

$$-a_{i1}x_1 - a_{i2}x_2 - \cdots - a_{in}x_n \leq -b_i.$$

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Every LP can be Transformed to Standard Form

- *linear equations*

Every LP can be Transformed to Standard Form

- *linear equations*

  The linear equation

$$a_{i1}x_i + \cdots + a_{in}x_n = b_i$$

can be written as two linear inequalities

$$a_{i1}x_1 + \cdots + a_{in}x_n \leq b_i$$

and

$$a_{i1}x_1 + \cdots + a_{in}x_n \geq b_i.$$

or equivalently

$$a_{i1}x_1 + \cdots + a_{in}x_n \leq b_i$$
$$-a_{i1}x_1 - \cdots - a_{in}x_n \leq -b_i$$

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Every LP can be Transformed to Standard Form

- *variables with lower bounds*

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- *variables with lower bounds*

  If a variable x_i has lower bound l_i which is not zero $(l_i \leq x_i)$ or equivalently, $0 \leq x_i - l_i$, one obtains a non-negative variable $w_i := x_i - l_i$ yielding the substitution
  $$x_i = w_i + l_i.$$
  In this case, the bound $l_i \leq x_i$ is equivalent to the bound $0 \leq w_i$.

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- *variables with lower bounds*

  If a variable x_i has lower bound l_i which is not zero ($l_i \leq x_i$) or equivalently, $0 \leq x_i - l_i$, one obtains a non-negative variable $w_i := x_i - l_i$ yielding the substitution

  $$x_i = w_i + l_i.$$

  In this case, the bound $l_i \leq x_i$ is equivalent to the bound $0 \leq w_i$.

- *variables with upper bounds*

  If a variable x_i has an upper bound u_i ($x_i \leq u_i$), or equivalently, $0 \leq u_i - x_i$, one obtains a non-negative variable $w_i := u_i - x_i$ yielding the substitution

  $$x_i = u_i - w_i.$$

  In this case, the bound $x_i \leq u_i$ is equivalent to the bound $0 \leq w_i$.

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Every LP can be Transformed to Standard Form

- *variables with interval bounds*

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  An interval bound of the form $l_i \leq x_i \leq u_i$ can be transformed into one non-negativity constraint and one linear inequality constraint in standard form by making the substitution

$$x_i = w_i + l_i.$$

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  An interval bound of the form $l_i \leq x_i \leq u_i$ can be transformed into one non-negativity constraint and one linear inequality constraint in standard form by making the substitution

$$x_i = w_i + l_i.$$

  In this case, the bounds $l_i \leq x_i \leq u_i$ are equivalent to the constraints

$$0 \leq w_i \qquad \text{and} \qquad w_i \leq u_i - l_i.$$

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Every LP can be Transformed to Standard Form

- *free variables*

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  Sometimes a variable is given without any bounds. Such variables are called free variables. To obtain standard form every free variable must be replaced by the difference of two non-negative variables. That is, if x_i is free, then we get

$$x_i = u_i - v_i$$

  with $0 \leq u_i$ and $0 \leq v_i$.

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Transformation to Standard Form

Put the following LP into standard form.

$$
\begin{array}{lrcrcrcrcr}
\text{minimize} & 3x_1 & - & x_2 & & & & & & \\
\text{subject to} & -x_1 & + & 6x_2 & - & x_3 & + & x_4 & \geq & -3 \\
 & & & 7x_2 & & & + & x_4 & = & 5 \\
 & & & & & x_3 & + & x_4 & \leq & 2
\end{array}
$$

$$-1 \leq x_2, \quad x_3 \leq 5, \quad -2 \leq x_4 \leq 2.$$

Transformation to Standard Form

The hardest part of the translation to standard form, or at least the part most susceptible to error, is the replacement of existing variables with non-negative variables.

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Reduce errors by doing the transformation in two steps.

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Reduce errors by doing the transformation in two steps.

Step 1: Make all of the changes that do not involve a variable substitution.

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Transformation to Standard Form

The hardest part of the translation to standard form, or at least the part most susceptible to error, is the replacement of existing variables with non-negative variables.

Reduce errors by doing the transformation in two steps.

Step 1: Make all of the changes that do not involve a variable substitution.

Step 2: Make all of the variable substitutions.

[18] Author: James Burke, University of Washington

Must be a maximization problem

$$\begin{array}{lrcrcrcrcr}
\text{minimize} & 3x_1 & - & x_2 & & & & & & \\
\text{subject to} & -x_1 & + & 6x_2 & - & x_3 & + & x_4 & \geq & -3 \\
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$$-1 \leq x_2, \quad x_3 \leq 5, \quad -2 \leq x_4 \leq 2.$$

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$$-1 \leq x_2, \quad x_3 \leq 5, \quad -2 \leq x_4 \leq 2.$$

Minimization $\Longrightarrow$ maximization

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Must be a maximization problem

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\begin{array}{lrcrcrcrcr}
\text{minimize} & 3x_1 & - & x_2 & & & & & & \\
\text{subject to} & -x_1 & + & 6x_2 & - & x_3 & + & x_4 & \geq & -3 \\
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\end{array}
$$

$$-1 \leq x_2, \quad x_3 \leq 5, \quad -2 \leq x_4 \leq 2.$$

Minimization $\Longrightarrow$ maximization

$$\text{maximize } -3x_1 + x_2.$$

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Inequalities must go the right way.

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$$-x_1 + 6x_2 - x_3 + x_4 \geq -3$$

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$$-x_1 + 6x_2 - x_3 + x_4 \geq -3$$

becomes

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Inequalities must go the right way.

$$-x_1 + 6x_2 - x_3 + x_4 \geq -3$$

becomes

$$x_1 - 6x_2 + x_3 - x_4 \leq 3.$$

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Equalities are replaced by 2 inequalities.

$$7x_2 + x_4 = 5$$

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becomes

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$$7x_2 + x_4 = 5$$

becomes

$$7x_2 + x_4 \leq 5$$

and

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Equalities are replaced by 2 inequalities.

$$7x_2 + x_4 = 5$$

becomes

$$7x_2 + x_4 \leq 5$$

and

$$-7x_2 - x_4 \leq -5$$

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Grouping upper bounds with the linear inequalities.

The double bound $-2 \leq x_4 \leq 2$ indicates that we should group the upper bound with the linear inequalities.

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$$x_4 \leq 2$$

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Step 1: Transformation to Standard Form

Combining all of these changes gives the LP

$$
\begin{array}{lrcrcrcrcr}
\text{maximize} & -3x_1 & + & x_2 & & & & & & \\
\text{subject to} & x_1 & - & 6x_2 & + & x_3 & - & x_4 & \leq & 3 \\
 & & & 7x_2 & & & + & x_4 & \leq & 5 \\
 & & - & 7x_2 & & & - & x_4 & \leq & -5 \\
 & & & & & x_3 & + & x_4 & \leq & 2 \\
 & & & & & & & x_4 & \leq & 2
\end{array}
$$

$$-1 \leq x_2, \quad x_3 \leq 5, \quad -2 \leq x_4.$$

Step 2: Variable Replacement

The variable x_1 is free, so we replace it by

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The variable x_1 is free, so we replace it by

$$x_1 = z_1^+ - z_1^- \text{ with } 0 \leq z_1^+, 0 \leq z_1^- .$$

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The variable x_1 is free, so we replace it by

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x_2 has a non-zero lower bound ($-1 \le x_2$) so we replace it by

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$$x_1 = z_1^+ - z_1^- \text{ with } 0 \leq z_1^+, 0 \leq z_1^-.$$

x_2 has a non-zero lower bound $(-1 \leq x_2)$ so we replace it by

$$z_2 = x_2 + 1 \quad \text{or} \quad x_2 = z_2 - 1 \quad \text{with} \quad 0 \leq z_2.$$

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x_3 is bounded above $(x_3 \le 5)$, so we replace it by

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x_3 is bounded above $(x_3 \le 5)$, so we replace it by

$$z_3 = 5 - x_3 \quad \text{or} \quad x_3 = 5 - z_3 \quad \text{with} \quad 0 \le z_3.$$

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x_3 is bounded above $(x_3 \leq 5)$, so we replace it by

$$z_3 = 5 - x_3 \quad \text{or} \quad x_3 = 5 - z_3 \quad \text{with} \quad 0 \leq z_3.$$

x_4 is bounded below $(-2 \leq x_4)$, so we replace it by

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The variable x_1 is free, so we replace it by

$$x_1 = z_1^+ - z_1^- \text{ with } 0 \leq z_1^+, 0 \leq z_1^- .$$

x_2 has a non-zero lower bound $(-1 \leq x_2)$ so we replace it by

$$z_2 = x_2 + 1 \quad \text{or} \quad x_2 = z_2 - 1 \quad \text{with} \quad 0 \leq z_2.$$

x_3 is bounded above $(x_3 \leq 5)$, so we replace it by

$$z_3 = 5 - x_3 \quad \text{or} \quad x_3 = 5 - z_3 \quad \text{with} \quad 0 \leq z_3.$$

x_4 is bounded below $(-2 \leq x_4)$, so we replace it by

$$z_4 = x_4 + 2 \quad \text{or} \quad x_4 = z_4 - 2 \quad \text{with} \quad 0 \leq z_4.$$

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Step 2: Transformation to Standard Form

Substituting $x_1 = z_1^+ - z_1^-$ into

$$\begin{array}{lrcrcrcrcr} \text{maximize} & -3x_1 & + & x_2 & & & & & & \\ \text{subject to} & x_1 & - & 6x_2 & + & x_3 & - & x_4 & \leq & 3 \\ & & & 7x_2 & & & + & x_4 & \leq & 5 \\ & & - & 7x_2 & & & - & x_4 & \leq & -5 \\ & & & & & x_3 & + & x_4 & \leq & 2 \\ & & & & & & & x_4 & \leq & 2 \end{array}$$

$$-1 \leq x_2, \quad x_3 \leq 5, \quad -2 \leq x_4.$$

gives

Step 2: Transformation to Standard Form

$$\begin{array}{lrcrcrcrcrcr}
\text{maximize} & -3z_1^+ & + & 3z_1^- & + & x_2 & & & & & & \\
\text{subject to} & z_1^+ & - & z_1^- & - & 6x_2 & + & x_3 & - & x_4 & \leq & 3 \\
& & & & & 7x_2 & & & + & x_4 & \leq & 5 \\
& & & & - & 7x_2 & & & - & x_4 & \leq & -5 \\
& & & & & & & x_3 & + & x_4 & \leq & 2 \\
& & & & & & & & & x_4 & \leq & 2
\end{array}$$

$$0 \leq z_1^+, \; 0 \leq z_1^-, \; -1 \leq x_2, \; x_3 \leq 5, \; -2 \leq x_4.$$

Step 2: Transformation to Standard Form

Substituting $x_2 = z_2 - 1$ into

$$\begin{array}{lrcrcrcrcrcr}
\text{maximize} & -3z_1^+ & + & 3z_1^- & + & x_2 & & & & & & \\
\text{subject to} & z_1^+ & - & z_1^- & - & 6x_2 & + & x_3 & - & x_4 & \leq & 3 \\
 & & & & & 7x_2 & & & + & x_4 & \leq & 5 \\
 & & & & - & 7x_2 & & & - & x_4 & \leq & -5 \\
 & & & & & & & x_3 & + & x_4 & \leq & 2 \\
 & & & & & & & & & x_4 & \leq & 2
\end{array}$$

$$0 \leq z_1^+, \; 0 \leq z_1^-, \; -1 \leq x_2, \; x_3 \leq 5, \; -2 \leq x_4.$$

gives

Step 2: Transformation to Standard Form

$$\begin{array}{lrcrcrcrcrll}
\text{maximize} & -3z_1^+ & + & 3z_1^- & + & z_2 & - & 1 & & & & \\
\text{subject to} & z_1^+ & - & z_1^- & - & 6z_2 & + & x_3 & - & x_4 & \leq & 3-6=-3 \\
 & & & & & 7z_2 & & & + & x_4 & \leq & 5+7=12 \\
 & & & & - & 7z_2 & & & - & x_4 & \leq & -5-7=-12 \\
 & & & & & & & x_3 & + & x_4 & \leq & 2 \\
 & & & & & & & & & x_4 & \leq & 2
\end{array}$$

$$0 \leq z_1^+, \ 0 \leq z_1^-, \ -0 \leq z_2, \ x_3 \leq 5, \ -2 \leq x_4.$$

Step 2: Transformation to Standard Form

$$\begin{array}{lrcrcrcrcrcr} \text{maximize} & -3z_1^+ & + & 3z_1^- & + & z_2 & & & & & & \\ \text{subject to} & z_1^+ & - & z_1^- & - & 6z_2 & + & x_3 & - & x_4 & \leq & -3 \\ & & & & & 7z_2 & & & + & x_4 & \leq & 12 \\ & & & & - & 7z_2 & & & - & x_4 & \leq & -12 \\ & & & & & & & x_3 & + & x_4 & \leq & 2 \\ & & & & & & & & & x_4 & \leq & 2 \end{array}$$

$$0 \leq z_1^+, \ 0 \leq z_1^-, \ -0 \leq z_2, \ x_3 \leq 5, \ -2 \leq x_4.$$

Step 2: Transformation to Standard Form

Substituting $x_3 = 5 - z_3$ into

$$\begin{array}{lrcrcrcrcrcr}
\text{maximize} & -3z_1^+ & + & 3z_1^- & + & z_2 & & & & & & \\
\text{subject to} & z_1^+ & - & z_1^- & - & 6z_2 & + & x_3 & - & x_4 & \leq & -3 \\
& & & & & 7z_2 & & & + & x_4 & \leq & 12 \\
& & & & - & 7z_2 & & & - & x_4 & \leq & -12 \\
& & & & & & & x_3 & + & x_4 & \leq & 2 \\
& & & & & & & & & x_4 & \leq & 2
\end{array}$$

$$0 \leq z_1^+,\ 0 \leq z_1^-,\ -0 \leq z_2,\ x_3 \leq 5,\ -2 \leq x_4.$$

gives

Step 2: Transformation to Standard Form

$$\begin{array}{lrcrcrcrcrcl}
\text{maximize} & -3z_1^+ & + & 3z_1^- & + & z_2 & & & & & & \\
\text{subject to} & z_1^+ & - & z_1^- & - & 6z_2 & - & z_3 & - & x_4 & \leq & -3-5=-8 \\
 & & & & & 7z_2 & & & + & x_4 & \leq & 12 \\
 & & & & - & 7z_2 & & & - & x_4 & \leq & -12 \\
 & & & & & & - & z_3 & + & x_4 & \leq & 2-5=-3 \\
 & & & & & & & & & x_4 & \leq & 2
\end{array}$$

$$0 \leq z_1^+,\ 0 \leq z_1^-,\ -0 \leq z_2,\ 0 \leq z_3,\ -2 \leq x_4.$$

Step 2: Transformation to Standard Form

Substituting $x_4 = z_4 - 2$ into

$$\begin{array}{lrcrcrcrcrcl}
\text{maximize} & -3z_1^+ & + & 3z_1^- & + & z_2 & & & & & & \\
\text{subject to} & z_1^+ & - & z_1^- & - & 6z_2 & - & z_3 & - & x_4 & \leq & -8 \\
& & & & & 7z_2 & & & + & x_4 & \leq & 12 \\
& & & & - & 7z_2 & & & - & x_4 & \leq & -12 \\
& & & & & & - & z_3 & + & x_4 & \leq & -3 \\
& & & & & & & & & x_4 & \leq & 2
\end{array}$$

$$0 \leq z_1^+, \; 0 \leq z_1^-, \; -0 \leq z_2, \; 0 \leq z_3, \; -2 \leq x_4.$$

gives

Step 2: Transformation to Standard Form

$$\begin{array}{lrcrcrcrcrl}
\text{maximize} & -3z_1^+ & + & 3z_1^- & + & z_2 & & & & & \\
\text{subject to} & z_1^+ & - & z_1^- & - & 6z_2 & - & z_3 & - & z_4 \leq & -8-2=-10 \\
 & & & & & 7z_2 & & & + & z_4 \leq & 12+2=14 \\
 & & & & - & 7z_2 & & & - & z_4 \leq & -12-2=-14 \\
 & & & & & & - & z_3 & + & z_4 \leq & -3+2=-1 \\
 & & & & & & & & & z_4 \leq & 2+2=4
\end{array}$$

$$0 \leq z_1^+,\ 0 \leq z_1^-,\ -0 \leq z_2,\ 0 \leq z_3,\ 0 \leq z_4.$$

which is in standard form.

Step 2: Transformation to Standard Form

After making these substitutions, we get the following LP in standard form:

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$$\begin{array}{lrcrcrcrcrcr}
\text{maximize} & -3z_1^+ & + & 3z_1^- & + & z_2 & & & & & & \\
\text{subject to} & z_1^+ & - & z_1^- & - & 6z_2 & - & z_3 & - & z_4 & \leq & -10 \\
 & & & & & 7z_2 & & & + & z_4 & \leq & 14 \\
 & & & & - & 7z_2 & & & - & z_4 & \leq & -14 \\
 & & & & & & - & z_3 & + & z_4 & \leq & -1 \\
 & & & & & & & & & z_4 & \leq & 4
\end{array}$$

$$0 \leq z_1^+, z_1^-, z_2, z_3, z_4.$$

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Transformation to Standard Form: Practice

Transform the following LP to an LP in standard form.

$$\begin{array}{lrcrcrcr}
\text{minimize} & x_1 & - & 12x_2 & + & 2x_3 & & \\
\text{subject to} & -5x_1 & - & x_2 & + & 3x_3 & = & -15 \\
 & 2x_1 & + & x_2 & - & 20x_3 & \geq & -30 \\
 & 0 \leq & x_2 & , & 1 \leq & x_3 & \leq 4 &
\end{array}$$