7 THINGS I WISH I HAD KNOWN AS A MATH MAJOR

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I have been a mathematician a long time now, but there was a time in my young adult life when I did not know being a mathematician was even a career possibility. I came into the profession a bit late perhaps. As a child, I had never met a mathematician. In fact, I didn't know there weren't many women in math historically until it was too late, I was already hooked. Perhaps that bit of naivety was good, but the rest of my naivety made some of the initial steps harder.

For those of you just entering the field, here is some insight and advice I wish I had known when I first uttered the words "well, I wish I could take another math class" to my advisor at the start of sophomore year of college. For the record, he was a professor in the architecture department and probably wasn't expecting to hear that, but he remembered it and circled back around to that comment over the course of our conversation. He helped me find my way to being a mathematician and for that I am very grateful.

(1) **Modern Mathematics is Growing Rapidly** Did you know, there exist unsolved problems in math? The first time I was told an unsolved problem in math was in my sophomore year of college. Prof. Gian-Carlo Rota's first lecture in probability was on 5 unsolved problems. I had no idea there was anything left to solve! If there were problems left to solve, then I vowed in my mind to go to grad school, get my Ph.D. in math, and work on solving some of them! It was a very powerful moment in my life.

We teach what we know, and so it makes sense that we don't teach what we don't know – very often. This gives students the false impression that "math is done, static, fixed for life". The reality is that our mathematical knowledge is alive and growing. Paul Erdös and Alfréd Rényi are quoted as saying "A mathematician is a machine that turns coffee into theorems". People prove new theorems every day and write them up hoping to publish them in professional journals. We advertise them to others in the field in conferences, and seminars, and over coffee, and dinner. That's what it means to do "research in math".

To give some perspective on how fast math is growing, consider the following statistics. According to the math arXiv which is a repository of prepublication manuscripts, they accepted 39,092 articles in 2020. For comparison, in 2000, they accepted 3093 articles, [`https://arxiv.org/year/math/`]

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One could say math has grown by an order of magnitude in twenty years, but that is not quite right. Back in 2000, the internet was still new and not every mathematician in the world was posting their papers to the arXiv. Now it is expected, if not required, of academic mathematicians to freely share their manuscripts on the arXiv in addition to publishing. Still, the number of papers published with new theorems each year has grown a lot, and the number of mathematicians has grown a lot in my lifetime. In the past 100 years, math has had a revolution, and it now infects every aspect of our lives, culture, business, government, and our future. For evidence of that, just check out any of the mathematical professional societies like the American Mathematical Society (AMS), the Society for Industrial and Applied Mathematics (SIAM), and the Mathematical Association of American (MAA).

We mathematicians are on our toes trying to learn new things throughout our lives. Those papers on the arXiv, the seminar talks, colloquia, conference, and workshops are about new math that was just developed in the past few years. It's going to take a while before any of it reaches into the classroom and the textbooks, but its coming. If you want to learn more about it, you can apply for an REU (Research Experience for Undergrads) or talk to a professor from a class you particularly liked. They may have a good project for you. Apply for an internship with a tech company or a government agency like the National Security Agency focusing on math or data science.

If you like the idea of solving some of the unsolved problems in math, then grad school in math is for you. To get a Ph.D., you need to prove a new publishable theorem, something no one in the world currently knows how to do.

If you like the idea of solving some unsolved problems in the real world, then applied math might be right for you, or computer science or mathematical biology, or electrical engineering, or going into business or government. There are lots of ways math is being used to solve some of the world's big problems. Being an applied mathematician is a great way to combine your math skills with your other interests.

It is a very exciting time to be a mathematician. New ideas are constantly emerging. We need brave, creative, intelligent, curious people like you to help us explore this new frontier. There is still so much to discover. And, there are good jobs out there for people with strong math skills. According to US News and World Reports, being a mathematician is #6 rated profession in Business Jobs, and being a statistician is #2, so combined we are dominating the rankings! For clarity, I do believe statisticians are mathematicians, and conversely. Statistics is a subdiscipline of math. But, then again, I think everything is part of math!

(2) **The secret to getting an A in a math class is . . . drum roll please . . . memorize the definitions and named theorems as soon as possible.** It's that simple! If a new vocabulary word is given on Monday, commit it to memory by Wednesday. If a named theorem like "Noether's First Isomorphism Theorem" has several adjectives in the hypothesis and 3 parts in the statement, make a flashcard. Recite the vocabulary for yourself as you are walking around town. Why? Because that is what the professors are expecting of you. They will go on to use those vocabulary words and theorems right away without much repetition. Mathematicians generally don't like to repeat themselves. It is part of the culture, which you may or may not like, but it is necessary in my humble opinion. Repetition takes time. If an instructor wants to cover a subject in-depth, it requires a pretty quick pace to state all of the necessary steps leading up to a big, useful result in one semester.

Help your instructor help you by putting in the work outside of class needed to keep up. Learning math is just like learning French. If you want to learn to speak French, you are going to have to learn to use the vocabulary. Practice helps, flashcards help, repetition helps, talking with others helps, listening to YouTube videos (Numberphiles), writing songs (Tom Lehrer), incorporating it into your short stories (Hermione's Group Theory) etc it all helps.

Back in elementary school and high school, you may have been bored while the math teacher explained the same concepts multiple times. Luckily, that won't happen again once you are past calculus! On the other hand, if you don't take the time to memorize the vocabulary, you will quickly be lost in class and bored again because you don't understand much. You may find that seriously frustrating while at the same time the instructor thinks they are going slowly and carefully.

(3) **Read ahead in the textbook.** People learn at different rates and in different styles. This diversity of learning styles is impossible to address entirely in the classroom. As apprentice mathematicians though, you should be honing your skills of independent learning by reading the textbook carefully. Read each assigned chapter. Better yet, look over each chapter before the lecture, go to the lecture, and come home and read the chapter again before attacking the homework.

"What, who has time for that?" You do! It is more efficient to learn the material when you read ahead. By looking at the textbook pages on a topic before the lecture on that topic, you will see what the key vocabulary words will be, you will see what the main theorems include. You will be better able to memorize them temporarily in the class, so you will understand the proofs better. The examples done in class will make more sense. You will be able to answer the instructors' questions, which are gifts to the prepared students given in order to help you digest the material. So, you will leave the lecture

in better shape to internalize the material just received in time for the next lecture, so the next lecture will go better. By induction, you will learn better in the end.

How do you feel when you are asked questions in class? To me, questions are helpful when they come at the right moment in my understanding. They can be alarming when I am behind and unsure about how to answer. Is that true for you too? If so, break the cycle of alarm and fear by starting the quarter/semester off right. Buy the textbook before the class starts. Skim the first chapter 3 days before the class starts. Get the syllabus on the first day of class and see what is coming next. Keep up with the lesson plans for the next lecture. It's ok to ask the professor what's next if they are skipping around in the book. They will understand if you explain that you need to read ahead to make the most of the lectures.

Textbooks in math are wonderful resources with very careful writing. Some are better than others. There are often many extra textbooks on the same subject in your library. Read the one assigned to the class, but if you find it frustrating for any reason, go to the library and find another one that suits you better. Maybe you need a book that is aimed at a lower level because yours is too terse, maybe you need one that has more pictures, maybe you want one that has more applications. Choose whichever one you like for whatever reason you like. By taking the initiative to look over the options in the library, you will learn more about the topic and be better prepared to use the material you are learning in class to solve real problems.

(4) **Professors are people too.** Talk to them — in person. Go to office hours. Ask questions. Ask for advice. Attack your homework early so that if you get stuck there is time to discuss it. Sometimes there is just a typo on the homework and it can be easily addressed. Sometimes there is a real misunderstanding in your thinking that can and should be clarified. The best way to figure it out is by having a conversation.

I'll admit that I used to hyperventilate a lot when talking to the senior, unapproachable, grumpy professor types. I could barely get my questions out. Luckily, they were more patient with me than I expected. Sometimes they offered me a chair in their office and sat in another chair nearby so we were on the same level. About 5 long minutes into the conversation, my breathing would return to normal. After that, the next conversation was a bit easier. The next got even easier. I am very grateful to the professors who waited while I caught my breath. They taught me how to do math, to write math, to study math, and to attack hard problems. I don't remember exactly what the problem was each of those times, but what came across was how they worked through the process. They helped me identify what word I misunderstood, or how I was only giving half a proof. Sometimes, they told me a bit more about

the math or about their lives. As an undergrad, I still treated them as scary aliens from another planet in some ways, but in other ways, they did become human in my mind.

As I got older, I got a few invitations to visit the homes of professors. I met their family, their dog, their mother, etc. I came to see them as humans who cared about education who had lives outside the classroom too. Seeing their human side, helped me see myself as a future mathematician. Turns out I'm one of them now, and I'm pretty sure I am still human. I hope my students can see that. I don't ever try to hide it, but teaching is a form of acting. We all put on our best face for the show. We have to put aside the fact that we just stubbed our toe, the car wouldn't start that morning and there was nothing in the fridge for breakfast because we forgot to buy eggs, or whatever life brings! Some professors focus more on the acting side and some focus more on the math side. Both are good, and both require some flexibility from the students to get through the important material together. Be patient and kind to the humans at your school, and they will return the favor.

(5) **Find a math friend.** Barbie is so right to say "Math is hard". It is hard. It just gets harder the more you learn right up to the point that it is so hard that no one in the whole world knows how to prove it. So how do we live with that? Answer: We work together.

I was very lucky to find a math friend in my sophomore year of college named Drew Sutherland. We got together regularly to discuss homework problems about a day before they were due. We were both very good students so we did the best we could on problems before our discussion, but there were just times when I missed something. Drew would help me straighten it out. I hope I helped him sometimes too. In grad school, I found even more math friends who helped me learn so much more deeply than I could have ever done on my own. I still like learning from my math friends. Now we call it "doing math," but it's still a lot of learning just like when I was a student.

(6) **Join a math activity.** Math classes are important, but there is a lot to learn about the profession outside of formal classes too. Try going to the Putnam prep sessions, do an REU, join the Math Club or the AWM Student chapter. If there is a seminar advertised on a topic that interests you, give it a try. Introduce yourself to someone there so you don't feel like a stranger. Don't be alarmed if you get lost in the discussion the first time or even the 10th time. Get what you can out of the environment, and bring some work to do if you get too lost so you aren't wasting your time. Try to pick up a few things to ask about after the seminar. Little information bits can build up into better understanding.

(7) **Just like they say on airplanes, put on your own oxygen mask before assisting others!** Have you thought about why they say that on airplanes? It's because there may not be anyone else who can help you if you pass out, but we might not think of that in an emergency.

Take care of yourself as you go through this journey to becoming a mathematician. Have fun! Stay strong. Find a way to enjoy the process. Be a part of the math club, but don't feel like you are solely responsible for its success. Be a good TA or tutor, but don't put in so much effort that you forget to do your own course work. There is a fine line between being a good community contributor and getting overwhelmed with responsibilities that take away time from classwork or research. You will be able to make a bigger impact on other humans in the future if you first focus on your own career development. Others will be ok on their own too. Trust them to take care of themselves. Don't even compare yourself with them. Do your own job.

Being a good undergrad math major is about exploring math beyond calculus. Find the part that inspires and excites you. Take another class on a topic if you have some drive to learn more. Ask the more approachable professors for suggestions of what to learn next. Talk to grad students and postdocs about their experiences if you can. Mentors need to be cultivated over time. It is your job to find your passion, set your own achievable goals, and explore how your talents can be usefully employed by others. But, you don't have to do it alone. Talk to the other humans around you at all stages of their career and learn what is going on and how to get the opportunity to do it at the next level whether you go into data science in industry, actuarial analysis, computer science, operations research, engineering, a government lab, math grad school, finance, medicine, law, origami, art, or anywhere else your math skills take you.

I hope these tips help you find your way in as a math major. Most of all, don't get frustrated if you initially feel like others know more. That is always true, no matter how good you are at anything. There is always someone who knows more about some aspect than you do. The goal is to improve your knowledge, your understanding, and your circle of experiences by participating in math-related activities.

If you have your own advice on how to get the most out of your undergrad experience as a math major, please let me know. I'm starting a collection of related essays. I'm also happy to get feedback on this essay, it's a work in progress.

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