

Revised Problem 7-21:

Prove the following about the groups in Problem 7-20.

For all even n , the groups in parts (a) and (c) are not isomorphic to the direct product of the indicated factors.

For odd $n > 1$, the groups in parts (a) and (c) are isomorphic to the direct product of the indicated factors.

For all $n > 1$, the groups in parts (b) and (d) are not isomorphic to the direct product of the indicated factors. *Your homework does not include part (d); it is included here only to provide you with the corrected statement of the problem.*

Added “W” problems; that is, you may use the following without giving a proof.

Problem 7-15: $\mathrm{SO}(2)$, $\mathrm{U}(1)$, and $\mathbb{S}^1$ are isomorphic as Lie groups.

Isomorphic groups have isomorphic centers.

With the exception of $\mathrm{SO}(2)$, every central element of every matrix group in mentioned in Problem 7-20 is a multiple of the identity element. *Although you don't need to include a proof of this in your homework, if you will be using Lie groups after this course, you should probably work through the proof at some point. Here's a hint: Consider the matrix A that acts on the left on other matrices by interchanging the first two rows and multiplying one of them by -1 . This matrix A , of appropriate dimension, is in all the matrix groups mentioned in 7-20 (except of course the 1×1 matrix groups), so central elements must commute with it. More generally, consider variations on A corresponding to other pairs of rows.*