Eigenvalues and Eigenvectors of Electrical Networks

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Contents

1 Introduction

1.1 Definitions

We construct an electrical network from a graph with a set of vertices V, a set of undirected edges $E = \{e_1, e_2, ...e_j\}$, and a set of conductivities along each of the edges $\gamma = \{\gamma(e_1), \gamma(e_2), ...\gamma(e_j)\}$. Note that conductivity is the inverse of resistance.

We divide the nodes into two groups, the boundary vertices, denoted ∂G or $V_B = \{b_1, b_2, ...b_k\}$, and the interior vertices, denoted int G or $I = \{i_1, i_2, ...i_j\}$.

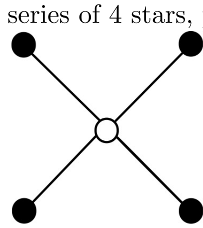
The boundary voltages are denoted f and the potential due to f is denoted u and is a function defined on the network which takes on the values of f at the boundary vertices. The current at the interior vertices is ϕ. The map from f to ϕ is the response map.

The Kirchhoff matrix, K, is defined by $\phi = Ku$ for both the interior and boundary vertices and is interpreted physically as the response current to voltages throughout the network. By manipulating the Kirchhoff matrix, one can find the response map for the network.

1.2 Electrical Networks, Past Results

Consider an electrical network inside of a black box. One can ask what the conductances of each of the edges of the graph are based on measurements of current and voltage at the nodes on the boundary (outside of the black box), called the inverse problem. One can also ask what the voltages and current at the boundary nodes are if there is some known conductivity at each of the edges in the graph, called the forward problem. This idea of being able to recover the conductivities of the edges of the network or being able to determine the voltages of the boundary from known conductivities inside of a black box is the subject of continued research and interest. Curtis and Morrow determine that resistors for rectangular networks can be recovered from measurements of boundary voltages as well as on circular networks.

Another relevant area of research for electrical networks is on n - to - 1 graphs. A graph is said to be n - to - 1 if there exists at most n distinct sets of conductivities which produce the same response matrix. In particular, researchers are interested in constructing graphs which are n - to - 1 where n is finite. Russell, and French examine 2^n - to - 1 graphs in particular. 2^n - to - 1 graphs are constructed from a series of 4 stars, pictured below.



In ongoing research, it is believed that pasting together 4-stars into different configurations can produce an n - to - 1 graph for any n in a systematic way.

As such, determining the properties of such a graph becomes important as this research develops. In particular, the eigenvalues of such a graph provide important information about current flow in an electrical network. The eigenvectors of an electrical network are values of the boundary voltage for which the boundary voltage is a scalar multiple of the boundary current. For a network with dozens of vertices, it can be difficult to predict how current will flow through the network. The eigenvalues are voltages for which the network will behave in a predictable way.

Cook finds the eigenvalues and eigenvectors for layered square lattice networks, tree networks, and ring networks. Willig and Wilson relate the composition of the response networks to its eigenvalues in addition to generating a general formula for finding the eigenvalues of a n-star (an n-star is a graph with n boundary vertices whose only connection is to a single, central interior vertex).

Additionally, I previously study the eigenvalues and eigenvectors of the same scaffold-type graph used in this paper. In my previous works, the conductivities of the edges of the graph are all equal to 1, and I find that the symmetry of this network produces eigenvalues and eigenvectors that are common among graphs whose number of vertices share factors.

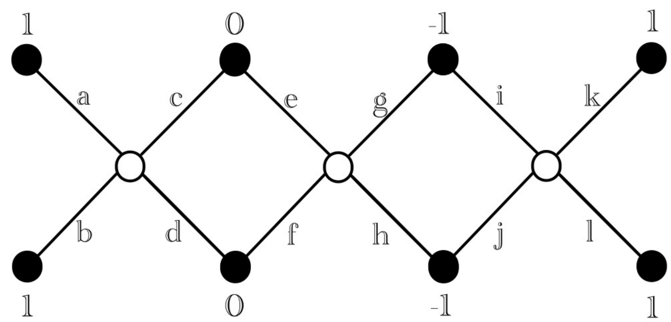
1.3 Research Question

This paper examines a series of pasted 4-stars and seeks to determine the eigenvalues and eigenvectors for different conductivities along vertices in the network. Specifically, this paper looks at zero and infinite resistances in a network and how eigenvectors and their corresponding eigenvalues might be used to determine which resistor(s) have the zero or infinite resistance.

2 Mathematical Background

2.1 Computations on Electrical Networks

Every interior vertex in the network obeys Kirchhoff's Law which states that for any node in a network where currents come together, the algebraic sum of the currents at that node must equal to zero. Additionally, we know that an electrical network is governed by Ohm's law which states that the voltage at any node in an electrical network is equal to the product of resistance and current, $V = I \cdot R$. Note that here V actually refers to the change in voltage where the change in voltage at some vertex, say V_i , is taken as the difference of the voltage at V_i and the voltage at its neighboring vertex. Since resistance is that multiplicative inverse of conductance, $I = \gamma \cdot V$. Given the electrical network below with boundary voltages and conductivities labelled, we can calculate the voltage (and eventually the current) at each boundary vertex.



Let V_{i_i} denote the voltage at the first interior vertex, V_{i_2} denote the voltage at the second interior vertex, and V_{i_3} denote the voltage at the third interior vertex. By Kirchhoff's law, we know that the net current at each interior vertex must be zero, so we have the following:

$$0 = (V_{i_1} - 1)a + (V_{i_1} - 1)b + (V_{i_1} - 0)c + (V - 0)d$$

$$V_{i_1} = \frac{a+b}{a+b+c+d}$$

$$0 = (V_{i_2} - 0)e + (V_{i_2} - 0)f + (V_{i_2} + 1)g + (V_{i_2} + 1)h$$

$$V_{i_2} = \frac{-g-h}{e+f+g+h}$$

$$0 = (V_{i_3} + 1)i + (V_{i_3} + 1)j + (V_{i_3} - 1)k + (V_{i_3} - 1)l$$

$$V_{i_3} = \frac{k+l-i-j}{i+j+k+l}$$

Now we find the current at each of the boundary vertices. Let I_{b_1} denote the current at the first boundary vertex, I_{b_2} denote the current at the second boundary vertex, and so on.

$$I_{b_1} = (1 - \frac{a+b}{a+b+c+d})a + (1 - \frac{k+l-i-j}{i+j+k+l})k$$

$$I_{b_2} = (1 - \frac{a+b}{a+b+c+d})b + (1 - \frac{k+l-i-j}{i+j+k+l})l$$

$$I_{b_3} = (0 - \frac{a+b}{a+b+c+d})c + (0 - \frac{-g-h}{e+f+g+h})e$$

$$I_{b_4} = (0 - \frac{a+b}{a+b+c+d})d + (0 - \frac{-g-h}{e+f+g+h})f$$

$$I_{b_5} = (-1 - \frac{-g-h}{e+f+g+h})g + (-1 - \frac{k+l-i-j}{i+j+k+l})i$$

$$I_{b_6} = (-1 - \frac{-g-h}{e+f+g+h})h + (-1 - \frac{k+l-i-j}{i+j+k+l})j$$

With these equations, we can find a way to algebraically determine the conductivities (if any) for which the boundary voltages of this electrical network are an eigenvector. In order to be an eigenvector, the voltage at each boundary vertex must be a scalar multiple of the boundary current. Let the scalar multiple be called λ, and the equations become:

$$I_{b_1} \cdot \lambda = 1 \cdot \lambda = (1 - \frac{a+b}{a+b+c+d})a + (1 - \frac{k+l-i-j}{i+j+k+l})k$$

$$I_{b_2} \cdot \lambda = 1 \cdot \lambda = (1 - \frac{a+b}{a+b+c+d})b + (1 - \frac{k+l-i-j}{i+j+k+l})l$$

$$I_{b_3} \cdot \lambda = 0 \cdot \lambda = (0 - \frac{a+b}{a+b+c+d})c + (0 - \frac{-g-h}{e+f+g+h})e$$

$$I_{b_4} \cdot \lambda = 0 \cdot \lambda = (0 - \frac{a+b}{a+b+c+d})d + (0 - \frac{-g-h}{e+f+g+h})f$$

$$I_{b_5} \cdot \lambda = -1 \cdot \lambda = (-1 - \frac{-g-h}{e+f+g+h})g + (-1 - \frac{k+l-i-j}{i+j+k+l})i$$

$$I_{b_6} \cdot \lambda = -1 \cdot \lambda = (-1 - \frac{-g-h}{e+f+g+h})h + (-1 - \frac{k+l-i-j}{i+j+k+l})j$$

Although informative as a calculation, this system of equations is not only nonlinear, but becomes increasingly difficult to solve as the network gets larger and larger. What is much more useful, is to be able to determine eigenvalues and eigenvectors from matrix algebra.

2.2 The Kirchhoff Matrix and the Response Matrix

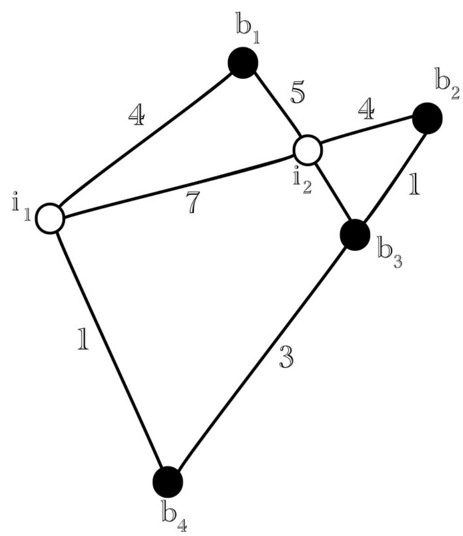
The Kirchhoff matrix is a tabulation of the conductivities between different vertices on the graph. If K is the Kirchhoff matrix and u is the voltage at the nodes of the network, then Ku is the resulting current in the network. It is divided into four quadrants, the first quadrant represent boundary to boundary connections, the second represents interior to boundary connections, the third is the transpose of the second, and the fourth represents interior to interior connections.

$$\begin{array}{c|cccccc|cccccc}
 & b_1 & b_2 & . & . & . & b_k & i_1 & i_2 & . & . & . & i_m \\
\hline
b_1 & & & & & & & & & & & & \\
b_2 & & & & & & & & & & & & \\
. & & & & & & & & & & & & \\
. & & & & & & & & & & & & \\
. & & & & & & & & & & & & \\
b_k & & & & & & & & & & & & \\
\hline
i_1 & & & & & & & & & & & & \\
i_2 & & & & & & & & & & & & \\
. & & & & & & & & & & & & \\
. & & & & & & & & & & & & \\
. & & & & & & & & & & & & \\
i_m, & & & & & & & & & & & &
\end{array}$$

To construct the matrix, we follow the following conventions: The Kirchhoff matrix is defined as follows where γ_{ij} is the conductivity between vertices v_i and v_j on the graph:

1. If no edge adjoins v_i and v_j, $\gamma_{ij} = 0$, Entries for which there is no connection between vertices have a value of zero.
2. For $i \neq j$, $K_{ij} = -\gamma_{ij}$, Off diagonal entries are given a negative sign.
3. For $i = j$, $K_{ij} = \sum_{j \neq i} \gamma_{ij}$, The diagonal entries are the negative of the column sum for their column after entering values into the non-diagonal entries.

Consider the following electrical network with conductivities labeled at each edge:



For this network, the Kirchhoff matrix would be the following:

	b_1	b_2	b_3	b_4	i_1	i_2
b_1	9	0	0	0	-4	-5
b_2	0	5	-1	0	0	-4
b_3	0	-1	5	-3	0	-1
b_4	0	0	-3	4	-1	0
i_1	-4	0	0	-1	12	-7
i_2	-5	-4	-1	0	-7	17

Now we assign names to the four quadrants of the Kirchhoff matrix. We call the fist quadrant A, the second B, the third B^T and the fourth C. In order to find the response matrix, Λ for the an electrical network, we find the Schur Complement of submatrix C of the Kirchhoff matrix which is equivalent to performing the following operations: $\Lambda = A - BC^{-1}B^T$. The justification for using this process to find the response matrix can be found in the text Inverse

Problems for Electrical Networks. For the example above, the response matrix is:

$$\Lambda = \begin{vmatrix} \frac{543}{155} & -\frac{352}{155} & -\frac{88}{155} & -\frac{103}{155} \\ -\frac{352}{155} & \frac{583}{155} & -\frac{203}{155} & -\frac{28}{155} \\ -\frac{88}{155} & -\frac{203}{155} & \frac{763}{155} & -\frac{472}{155} \\ -\frac{103}{155} & -\frac{28}{155} & -\frac{472}{155} & \frac{603}{155} \end{vmatrix}$$

With the response matrix, we can simply find the characteristic polynomial and calculate the eigenvalues for the network.

3 Results, A Single Dysfunctional Resistor

The eigenvalues and eigenvectors of electrical networks can be used to determine the cause of an open or of a short circuit. In an open circuit, there is zero conductivity between two nodes whereas a short circuit has infinite conductivity between two nodes.

3.1 Open Circuits

Eigenvalues: $\left\{0, 2, 2, \frac{1}{12}(15+\sqrt{33}), \frac{1}{12}(15-\sqrt{33}), \frac{3}{2}\right\}$

Conductivities: $\{0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix} \begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix} \begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix} \begin{bmatrix}\frac{1}{4}(-3+\sqrt{33})\\ \frac{1}{4}(-5-\sqrt{33}\\1\\1\\0\\0\end{bmatrix} \begin{bmatrix}\frac{1}{4}(-3-\sqrt{33})\\ \frac{1}{4}(-5+\sqrt{33}\\1\\1\\0\\0\end{bmatrix} \begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\1\\1\end{bmatrix} \tag{1}$$

Conductivities: $\{1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix} \begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix} \begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix} \begin{bmatrix}\frac{1}{4}(-5-\sqrt{33}\\ \frac{1}{4}(-3+\sqrt{33})\\1\\1\\0\\0\end{bmatrix} \begin{bmatrix}\frac{1}{4}(-5+\sqrt{33}\\ \frac{1}{4}(-3-\sqrt{33})\\1\\1\\0\\0\end{bmatrix} \begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\1\\1\end{bmatrix} \tag{2}$$

Conductivities: $\{1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}-\frac{-39-7\sqrt{33}}{9+\sqrt{33}}\\-\frac{-39-7\sqrt{33}}{9+\sqrt{33}}\\-6-\sqrt{33}\\1\\0\\0\end{bmatrix}\begin{bmatrix}-\frac{39-7\sqrt{33}}{-9+\sqrt{33}}\\-\frac{39-7\sqrt{33}}{-9+\sqrt{33}}\\-6+\sqrt{33}\\1\\0\\0\end{bmatrix}\begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\1\\1\end{bmatrix}\tag{3}$$

Conductivities: $\{1,1,1,0,1,1,1,1,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}-\frac{-1+\sqrt{33}}{-9+\sqrt{33}}\\-\frac{-1+\sqrt{33}}{-9+\sqrt{33}}\\\frac{1}{3}(-6-\sqrt{33})\\1\\0\\0\end{bmatrix}\begin{bmatrix}-\frac{1+\sqrt{33}}{9+\sqrt{33}}\\-\frac{1+\sqrt{33}}{9+\sqrt{33}}\\\frac{1}{3}(-6+\sqrt{33})\\1\\0\\0\end{bmatrix}\begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\1\\1\end{bmatrix}\tag{4}$$

Conductivities: $\{1,1,1,1,0,1,1,1,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\-\frac{-135+23\sqrt{33}}{-59+11\sqrt{33}}\\-\frac{17-\sqrt{33}}{-59+11\sqrt{33}}\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\-\frac{135+23\sqrt{33}}{59+11\sqrt{33}}\\-\frac{-17-\sqrt{33}}{59+11\sqrt{33}}\\1\\1\end{bmatrix}\begin{bmatrix}-2\\-2\\1\\1\\1\\1\end{bmatrix}\tag{5}$$

Conductivities: $\{1,1,1,1,1,0,1,1,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\-\frac{69+11\sqrt{33}}{9+7\sqrt{33}}\\-\frac{3(-17+\sqrt{33})}{9+7\sqrt{33}}\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\-\frac{-69+11\sqrt{33}}{-9+7\sqrt{33}}\\-\frac{3(-17+\sqrt{33})}{-9+7\sqrt{33}}\\1\\1\end{bmatrix}\begin{bmatrix}-2\\-2\\1\\1\\1\\1\end{bmatrix}\tag{6}$$

Conductivities: $\{1,1,1,1,1,1,0,1,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\-\frac{6(-211+35\sqrt{33})}{(-9+\sqrt{33})(9+7\sqrt{33})}\\-\frac{6(-211+35\sqrt{33})}{(-9+\sqrt{33})(9+7\sqrt{33})}\\-\frac{3(-59+11\sqrt{33})}{9+7\sqrt{33}}\\1\end{bmatrix}\begin{bmatrix}0\\0\\-\frac{6(211+35\sqrt{33})}{(9+\sqrt{33})(-9+7\sqrt{33})}\\-\frac{6(211+35\sqrt{33})}{(9+\sqrt{33})(-9+7\sqrt{33})}\\-\frac{3(59+11\sqrt{33})}{-9+7\sqrt{33}}\\1\end{bmatrix}\begin{bmatrix}-2\\-2\\1\\1\\1\\1\end{bmatrix} \tag{7}$$

Conductivities: $\{1,1,1,1,1,1,1,0,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\-\frac{2(-211+35\sqrt{33})}{(-9+\sqrt{33})(-59+11\sqrt{33})}\\-\frac{2(-211+35\sqrt{33})}{(-9+\sqrt{33})(-59+11\sqrt{33})}\\-\frac{9+7\sqrt{33}}{3(-59+11\sqrt{33})}\\1\end{bmatrix}\begin{bmatrix}0\\0\\-\frac{2(211+35\sqrt{33})}{(9+\sqrt{33})(59+11\sqrt{33})}\\-\frac{2(211+35\sqrt{33})}{(9+\sqrt{33})(59+11\sqrt{33})}\\-\frac{-9+7\sqrt{33}}{3(59+11\sqrt{33})}\\1\end{bmatrix}\begin{bmatrix}-2\\-2\\1\\1\\1\\1\end{bmatrix} \tag{8}$$

Conductivities: $\{1,1,1,1,1,1,1,1,0,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}-\frac{6(-211+35\sqrt{33})}{(-9+\sqrt{33})(9+7\sqrt{33})}\\-\frac{6(-211+35\sqrt{33})}{(-9+\sqrt{33})(9+7\sqrt{33})}\\0\\0\\-\frac{3(-59+11\sqrt{33})}{9+7\sqrt{33}}\\1\end{bmatrix}\begin{bmatrix}-\frac{6(211+35\sqrt{33})}{(9+\sqrt{33})(-9+7\sqrt{33})}\\-\frac{6(211+35\sqrt{33})}{(9+\sqrt{33})(-9+7\sqrt{33})}\\0\\0\\-\frac{3(59+11\sqrt{33})}{-9+7\sqrt{33}}\\1\end{bmatrix}\begin{bmatrix}1\\1\\-2\\-2\\1\\1\end{bmatrix} \tag{9}$$

Conductivities: $\{1,1,1,1,1,1,1,1,1,0,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}-\frac{2(-211+35\sqrt{33})}{(-9+\sqrt{33})(-59+11\sqrt{33})}\\-\frac{2(-211+35\sqrt{33})}{(-9+\sqrt{33})(-59+11\sqrt{33})}\\0\\0\\-\frac{9+7\sqrt{33}}{3(-59+11\sqrt{33})}\\1\end{bmatrix}\begin{bmatrix}-\frac{2(211+35\sqrt{33})}{(9+\sqrt{33})(59+11\sqrt{33})}\\-\frac{2(211+35\sqrt{33})}{(9+\sqrt{33})(59+11\sqrt{33})}\\0\\0\\-\frac{-9+7\sqrt{33}}{3(59+11\sqrt{33})}\\1\end{bmatrix}\begin{bmatrix}1\\1\\-2\\-2\\1\\1\end{bmatrix} \tag{10}$$

Conductivities: $\{1,1,1,1,1,1,1,1,1,1,0,1\}$

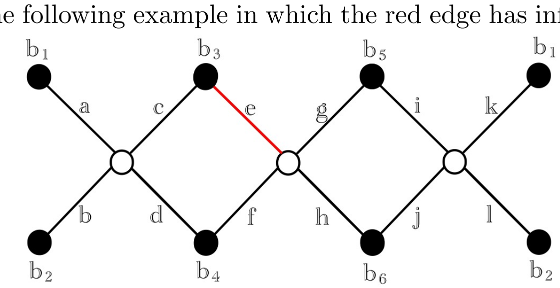
$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}-\frac{-135+23\sqrt{33}}{-59+11\sqrt{33}}\\-\frac{17-\sqrt{33}}{-59+11\sqrt{33}}\\0\\0\\1\\1\end{bmatrix}\begin{bmatrix}-\frac{135+23\sqrt{33}}{59+11\sqrt{33}}\\-\frac{-17-\sqrt{33}}{59+11\sqrt{33}}\\0\\0\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\-2\\-2\\1\\1\end{bmatrix} \tag{11}$$

Conductivities: $\{1,1,1,1,1,1,1,1,1,1,1,0\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}-\frac{69+11\sqrt{33}}{9+7\sqrt{33}}\\-\frac{3(-17+\sqrt{33})}{9+7\sqrt{33}}\\0\\0\\1\\1\end{bmatrix}\begin{bmatrix}-\frac{-69+11\sqrt{33}}{-9+7\sqrt{33}}\\-\frac{3(17+\sqrt{33})}{-9+7\sqrt{33}}\\0\\0\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\-2\\-2\\1\\1\end{bmatrix} \tag{12}$$

Note that for the network corresponding to each conductivity set, there are two unique eigenvectors. In order to determine which edge on the network has infinite resistance, one can apply the unique set of voltages given by the eigenvectors listed above. Then, after applying each voltage, one can measure the current through each vertex to determine if the current is the corresponding scalar multiple of the voltage.

Consider the following example in which the red edge has infinite resistance:



In order to determine which of the resistors in the network is out, we can systematically apply voltages at each of the boundary vertices. Then we measure the resulting current. Without knowing which edge has infinite resistance, we would take a unique eigenvector from the set which corresponds to the first edge having infinite resistance, apply the boundary voltages of that eigenvector, and then determine if the current at the boundary vertices was a scalar multiple of the voltage, and further, that the scalar multiple was equal to the eigenvalue which is assigned to the eigenvalue chosen.

We would follow the same process for a unique eigenvector from the set of eigenvectors for the network where the second edge had infinite resistance, then I would do the same for the network where the third edge had infinite resistance, and so on. In the case above, the fifth edge has infinite resistance; thus, we select an eigenvalue that is unique to the case where the fifth edge has infinite resistance.

The following eigenvector is unique to this network configuration, $\left\{0, 0, -\frac{-135+23\sqrt{33}}{-59+11\sqrt{33}}, -\frac{17-\sqrt{33}}{-59+11\sqrt{33}}, 1, 1\right\}$, and its related eigenvalue is $\frac{1}{12}(15 + \sqrt{33})$. In order to be sure that the fifth resistor is malfunctioning, I would apply a voltage of 0 at the first boundary vertex, 0 at the second, $-\frac{-135+23\sqrt{33}}{-59+11\sqrt{33}}$ at the third, $-\frac{17-\sqrt{33}}{-59+11\sqrt{33}}$ at the fourth, 1 at the fifth, and 1 at the sixth. If the fifth edge has infinite resistance, then the current at each vertex will be the voltage applied multiplied by $\frac{1}{12}(15 + \sqrt{33})$. The resulting currents if the fifth edge had zero resistance would then be:
$\left\{0, 0, \frac{1}{12}(15 + \sqrt{33}) \cdot -\frac{-135+23\sqrt{33}}{-59+11\sqrt{33}}, \frac{1}{12}(15 + \sqrt{33}) \cdot -\frac{17-\sqrt{33}}{-59+11\sqrt{33}}, \frac{1}{12}(15 + \sqrt{33}), \frac{1}{12}(15 + \sqrt{33})\right\}$

The result generalizes to networks with more vertices as well. For the network with 12 boundary vertices, the eigenvalues are: $\left\{2, 2, 2, 2, \frac{1}{6}(6 + \sqrt{6}), \frac{1}{6}(6 - \sqrt{6}), 0\right\}$.

We do not observe the same for networks where more than one of the edges has infinite resistance. Consider again the case with 12 boundary vertices, for example, when edge a is assigned conductivity 0 with other edges being assigned to zero conductivity in turn, the eigenvalues are:

Conductivities: $\{0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$
Eigenvalues: $\left\{2, 2, \frac{3}{2}, 1, \frac{1}{2}, 0\right\}$
Conductivities: $\{0, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$
Eigenvalues: $\left\{2, \frac{1}{4}(5 + \sqrt{5}), \frac{3}{2}, 1, \frac{1}{4}(5 - \sqrt{5}), 0\right\}$
Conductivities: $\{0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1\}$
Eigenvalues: $\left\{2, \frac{1}{4}(5 + \sqrt{5}), \frac{3}{2}, 1, \frac{1}{4}(5 - \sqrt{5}), 0\right\}$
Conductivities: $\{0, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1\}$
Eigenvalues: $\left\{2, \frac{1}{5}(7 + \sqrt{13}), \frac{5}{3}, 1, \frac{1}{4}(7 - \sqrt{13}), 0\right\}$
Conductivities: $\{0, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 1\}$
Eigenvalues: $\left\{2, \frac{1}{5}(7 + \sqrt{13}), \frac{5}{3}, 1, \frac{1}{4}(7 - \sqrt{13}), 0\right\}$
Conductivities: $\{0, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1\}$
Eigenvalues: $\left\{2, \frac{1}{5}(7 + \sqrt{13}), \frac{5}{3}, 1, \frac{1}{4}(7 - \sqrt{13}), 0\right\}$
Conductivities: $\{0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1\}$
Eigenvalues: $\left\{2, \frac{1}{5}(7 + \sqrt{13}), \frac{5}{3}, 1, \frac{1}{4}(7 - \sqrt{13}), 0\right\}$
Conductivities: $\{0, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1\}$
Eigenvalues: $\left\{2, \frac{1}{5}(7 + \sqrt{13}), \frac{5}{3}, 1, \frac{1}{4}(7 - \sqrt{13}), 0\right\}$
Conductivities: $\{0, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1\}$
Eigenvalues: $\left\{2, \frac{1}{5}(7 + \sqrt{13}), \frac{5}{3}, 1, \frac{1}{4}(7 - \sqrt{13}), 0\right\}$
Conductivities: $\{0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1\}$
Eigenvalues: $\left\{2, 2, \frac{5}{3}, \frac{4}{3}, 0, 0\right\}$
Conductivities: $\{0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0\}$
Eigenvalues: $\left\{2, 2, \frac{1}{3}(3 + \sqrt{3}), 1, \frac{1}{3}(3 - \sqrt{3}), 0\right\}$

Certainly there are patterns in the eigenvalues depending on where the second zero conductivity is in relation to edge a, but finding the dysfunctional resistor becomes much more complicated as we introduce more than one zero conductivity into the network. Additionally, in the above cases, we fix the conductivity of edge a at zero and assign each of the other edges to zero in turn, but as soon as we fix the conductivity of another edge, we find further variations in the eigenvalues. It seems unreasonable to use the above strategy to be able to determine which two (or more) resistors in a network are malfunctioning.

3.2 Short Circuits

We observe similar results for the case when one of the edges of a network has zero resistance. An edge with zero resistance has an infinite conductivity, but we can apply a similar process similar to the above for determining the cause of a short circuit. We simply let the conductivity of an edge go to infinity instead of setting the conductivity of an edge equal to zero. We find that the eigenvalues for a network with a single edge with zero resistance approach the following values: $\left\{0, \frac{3}{2}, 2, 2, \frac{1}{4}(13-\sqrt{41}), \frac{1}{4}(13+\sqrt{41})\right\}$. Using the same process as in the case of open circuits, we can use the following eigenvalues to determine the edge of the graph with infinite conductance. In each of the following sets, let x be a value approaching infinity.

Conductivities: $\{x,1,1,1,1,1,1,1,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}\frac{1}{4}(-7+\sqrt{41})\\\frac{1}{4}(-1-\sqrt{41})\\1\\1\\0\\0\end{bmatrix}\begin{bmatrix}\frac{1}{4}(-7-\sqrt{41})\\\frac{1}{4}(-1+\sqrt{41})\\1\\1\\0\\0\end{bmatrix} \tag{13}$$

Conductivities: $\{1,x,1,1,1,1,1,1,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}\frac{1}{4}(-1-\sqrt{41})\\\frac{1}{4}(-7+\sqrt{41})\\1\\1\\0\\0\end{bmatrix}\begin{bmatrix}\frac{1}{4}(-1+\sqrt{41})\\\frac{1}{4}(-7-\sqrt{41})\\1\\1\\0\\0\end{bmatrix} \tag{14}$$

Conductivities: $\{1,1,x,1,1,1,1,1,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}\frac{1}{10}(1-\sqrt{41})\\\frac{1}{10}(1-\sqrt{41})\\\frac{1}{5}(-6+\sqrt{41})\\1\\0\\0\end{bmatrix}\begin{bmatrix}\frac{1}{10}(1+\sqrt{41})\\\frac{1}{10}(1+\sqrt{41})\\\frac{1}{5}(-6-\sqrt{41})\\1\\0\\0\end{bmatrix} \tag{15}$$

Conductivities: $\{1,1,1,x,1,1,1,1,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\-\frac{1}{2}\\1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}\frac{1}{2}(-7-\sqrt{41})\\\frac{1}{2}(-7-\sqrt{41})\\\frac{1}{5}(6+\sqrt{41})\\1\\0\\0\end{bmatrix}\begin{bmatrix}\frac{1}{2}(-7+\sqrt{41})\\\frac{1}{2}(-7+\sqrt{41})\\6-\sqrt{41}\\1\\0\\0\end{bmatrix}\tag{16}$$

Conductivities: $\{1,1,1,1,x,1,1,1,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-2\\-2\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}0\\0\\\frac{1}{4}(-7+\sqrt{41})\\\frac{1}{4}(-1-\sqrt{41})\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\\frac{1}{4}(-7-\sqrt{41})\\\frac{1}{4}(-1+\sqrt{41})\\1\\1\end{bmatrix}\tag{17}$$

Conductivities: $\{1,1,1,1,1,x,1,1,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-2\\-2\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}0\\0\\\frac{1}{4}(-1-\sqrt{41})\\\frac{1}{4}(-7+\sqrt{41})\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\\frac{1}{4}(-1+\sqrt{41})\\\frac{1}{4}(-7-\sqrt{41})\\1\\1\end{bmatrix}\tag{18}$$

Conductivities: $\{1,1,1,1,1,1,x,1,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-2\\-2\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\\frac{1}{10}(1-\sqrt{41})\\\frac{1}{10}(1-\sqrt{41})\\\frac{1}{5}(-6+\sqrt{41})\\1\end{bmatrix}\begin{bmatrix}0\\0\\\frac{1}{10}(1+\sqrt{41})\\\frac{1}{10}(1+\sqrt{41})\\\frac{1}{5}(-6-\sqrt{41})\\1\end{bmatrix}\tag{19}$$

Conductivities: $\{1,1,1,1,1,1,1,x,1,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-2\\-2\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\\frac{1}{2}(-7-\sqrt{41})\\\frac{1}{2}(-7-\sqrt{41})\\6+\sqrt{41}\\1\end{bmatrix}\begin{bmatrix}0\\0\\\frac{1}{2}(-7+\sqrt{41})\\\frac{1}{2}(-7+\sqrt{41})\\6-\sqrt{41}\\1\end{bmatrix}\tag{20}$$

Conductivities: $\{1,1,1,1,1,1,1,1,x,1,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\-2\\-2\\1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}\frac{1}{10}(1-\sqrt{41})\\\frac{1}{10}(1-\sqrt{41})\\0\\0\\\frac{1}{5}(-6+\sqrt{41})\\1\end{bmatrix}\begin{bmatrix}\frac{1}{10}(1+\sqrt{41})\\\frac{1}{10}(1+\sqrt{41})\\0\\0\\\frac{1}{5}(-6-\sqrt{41})\\1\end{bmatrix} \tag{21}$$

Conductivities: $\{1,1,1,1,1,1,1,1,1,x,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\-2\\-2\\1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}\frac{1}{2}(-7-\sqrt{41})\\\frac{1}{2}(-7-\sqrt{41})\\0\\0\\6+\sqrt{41}\\1\end{bmatrix}\begin{bmatrix}\frac{1}{2}(-7+\sqrt{41})\\\frac{1}{2}(-7+\sqrt{41})\\0\\0\\6-\sqrt{41}\\1\end{bmatrix} \tag{22}$$

Conductivities: $\{1,1,1,1,1,1,1,1,1,1,x,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\-2\\-2\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}\frac{1}{4}(-7+\sqrt{41})\\\frac{1}{4}(-1-\sqrt{41})\\0\\0\\1\\1\end{bmatrix}\begin{bmatrix}\frac{1}{4}(-7-\sqrt{41})\\\frac{1}{4}(-1+\sqrt{41})\\0\\0\\1\\1\end{bmatrix} \tag{23}$$

Conductivities: $\{1,1,1,1,1,1,1,1,1,x,1,1\}$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\-2\\-2\\1\\1\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}\frac{1}{2}(-7-\sqrt{41})\\\frac{1}{2}(-7-\sqrt{41})\\0\\0\\6+\sqrt{41}\\1\end{bmatrix}\begin{bmatrix}\frac{1}{2}(-7+\sqrt{41})\\\frac{1}{2}(-7+\sqrt{41})\\0\\0\\6-\sqrt{41}\\1\end{bmatrix} \tag{24}$$

Conductivities: $\{1,1,1,1,1,1,1,1,1,1,1,x\}$

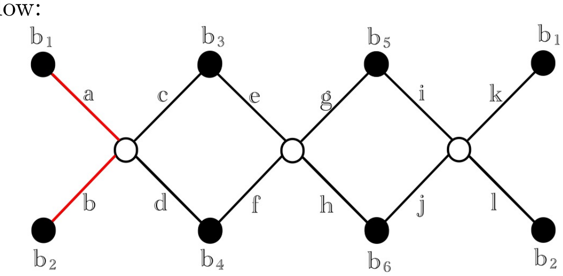
$$\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\-2\\-2\\1\\1\end{bmatrix}\begin{bmatrix}0\\0\\-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\-1\\1\end{bmatrix}\begin{bmatrix}\frac{1}{4}(-1-\sqrt{41})\\\frac{1}{4}(-7+\sqrt{41})\\0\\0\\1\\1\end{bmatrix}\begin{bmatrix}\frac{1}{4}(-1+\sqrt{41})\\\frac{1}{4}(-7-\sqrt{41})\\0\\0\\1\\1\end{bmatrix} \tag{25}$$

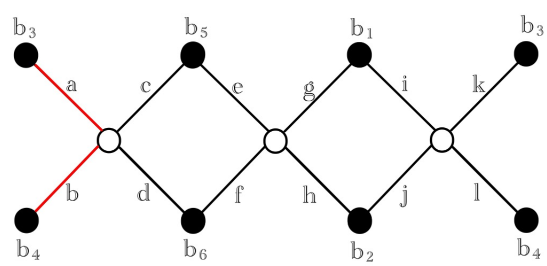
As was the case for open circuits, this process generalizes to networks with more boundary vertices. Additionally, like the open circuit, allowing more than one conductivity to be infinite breaks down the process of finding the dysfunctional resistor because the eigenvalues for the network depend on where the infinite conductances are. This case is further complicated by the fact that depending on where the infinite conductances are placed, infinite eigenvalues can result. Situations with multiple dysfunctional resistors are considered in more detail in the following section.

3.3 Explanation

We observe the same eigenvalues no matter which resistor in the network is assigned a value of zero. We can understand why this happens by considering the first two cases, where edge a (connected to b_1) is assigned a 0 conductivity and where the edge connected to b_2 is assigned a zero conductivity. Both b_1 and b_2 have the same interior connections. That is, they are connected to i_1 and i_3 in the case where there are three interior vertices (or i_n in the case where there are n interior vertices). Thus, b_1 and b_2 are identical up to the naming of the vertices and we expect identical eigenvalues.

Similarly, b_3 and b_1 both have 2 connections to other vertices, b_1 to i_1 and i_3 in the six vertex case, and b_3 to i_1 and i_2. Furthermore, regardless of whether the conductivity equals zero for the edge c or edge a, b_1 and b_3 both have a connection to one interior vertex where the voltage is determined by four conductivities with a value of 1 and one interior vertex where the voltage is determined by three conductivities with values of 1 and one conductivity with a value of zero. Thus, we expect to see the same eigenvalues for b_1 and b_3. We can see that b_1 and b_3 are identical up to the naming of the vertices on the illustration below:





The same reasoning is easily extended to show that the eigenvalues will be the same regardless of which edge has conductivity equal to zero on the graph with three interior vertices and on the graph with n interior vertices (although adding additional interior vertices changes the eigenvalues from the case with $n-1$ . . . 1 interior vertices). Additionally, the same reasoning holds for the short circuit. The eigenvalues for the short circuit will be the same regardless of which edge has infinite conductivity.

4 Results, Multiple Dysfunctional Resistor

In order to address the situation in which a network has multiple dysfunctional resistors, I found the eigenvalues for each electrical network with six boundary vertices with two, three, and four zeros along each edge. When there were two zero conductivities along the edges, there were five distinct sets of eigenvalues. When there were three zero conductances along the edges, there were nine distinct sets of eigenvalues. Finally, when there were four zero conductances along the edges, there were fifteen distinct eigenvalues.

Upon inspection, it became clear that networks that shared common eigenvalues also shared other common properties. The properties of the networks in each grouping are given in the following tables. These results are for the network with six boundary vertices. Note that it was only necessary to test configurations of networks with an edge with zero conductance in the first position because the symmetry of the network ensures repetition after the testing the configurations with the zero conductance in the first position.

The following conventions are used in the tables below:

BV: the number of boundary vertices connected to an edge with zero conductance

IV: the number of interior vertices connected to an edge with zero conductance

R1: the degrees of the interior vertices, considering only connections to edges with zero conductance

G1: the degrees of the interior vertices, considering only connections to edges with nonzero conductance

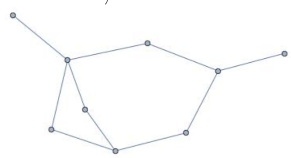
R2: the degrees of the interior vertices, considering only connections to edges with zero conductance that have a path length of two or more

G2: the degrees of the interior vertices, considering only connections to edges with nonzero conductance that have a path length of two or more

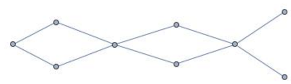
2 Zero Conductivities

Eigenvalue	*BV*	*IV*	*R1*	*G1*	*R2*	*G2*	*Total*	*Radius*	*Diameter*	*Mean Graph Distance*
1	2	2	1(1)	4(3)(3)	0	3	6	3	4	2.167
2	2	1	2	4(4)(2)	0	4	1	3	5	2.333
3	2	1	2	4(4)(2)	0	3	1	3	4	2.111
4	1	2	1(1)	4(3)(3)	1	4	1	3	3	1.821
5	2	2	1(1)	4(3)(3)	0	4	1	3	6	2.444

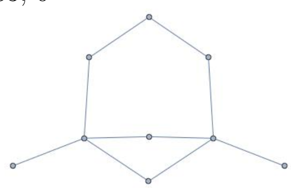
1. 2, 1.76759, 1.667, 1, 0.565741, 0



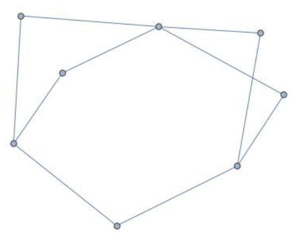
2. 2, 2, 1.5, 1, 0.5, 0



3. 2, 1.890902, 1.5, 0.690983, 0



4. 2, 2, 1.67, 1.33, 0, 0

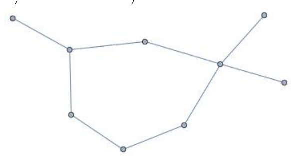


5. 2, 2, 1.57735, 1.042265, 0

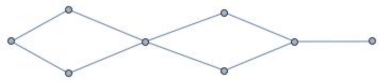
3 Zero Conditivities

Eigenvalue	*BV*	*IV*	*R1*	*G1*	*R2*	*G2*	*Total*	*Radius*	*Diameter*	*Mean Graph Distance*
1	3	2	2(1)	4(3)(2)	0	2	12	3	4	2.278
2	2	2	2(1)	4(3)(2)	1	4	3	3	5	2.179
3	3	2	2(1)	4(3)(2)	0	3	6	3	6	2.556
4	2	2	2(1)	4(3)(2)	1	3	6	3	4	1.964
5	3	2	2(1)	4(3)(2)	0	3	9	3	6, 5	2.667, 2.556, 2.5
6	3	2	1(1)(1)	3(3)(3)	0	3	6	3	6	2.722
7	2	3	1(1)(1)	3(3)(3)	1	3	6	3	4	2.036
8	3	3	1(1)(1)	3(3)(3)	0	2	4	3	4	2.333
9	3	1	3	4(4)(1)	0	3	3	3	5	2.389

1. 1.82116, 1.69206, 1, 1, 0.486772, 0

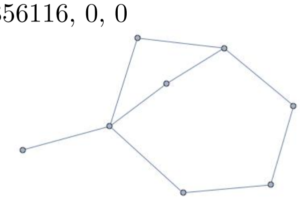


2. 2, 2, 1.40825, 0.591752, 0, 0

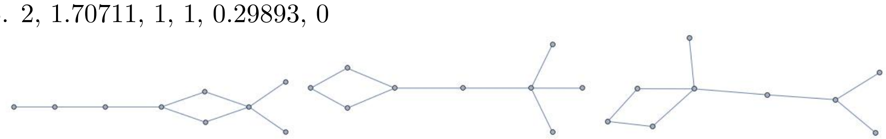


3. 2, 1.66995, 1.30469, 0.693514, 0.330050

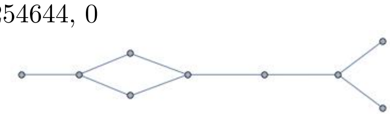
4. 2, 1.76567, 1.37821, 0.856116, 0, 0



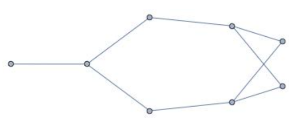
5. 2, 1.70711, 1, 1, 0.29893, 0



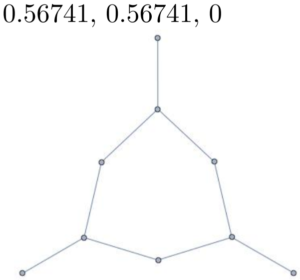
6. 2, 1.74536, 1, 1, 0.254644, 0



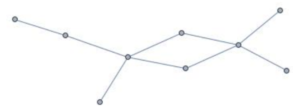
7. 2, 1.67, 1.67, 0.67, 0, 0



8. 1.76759, 1.76759, 1.33, 0.56741, 0.56741, 0



9. 2, 1.5, 1, 1, 0.5, 0

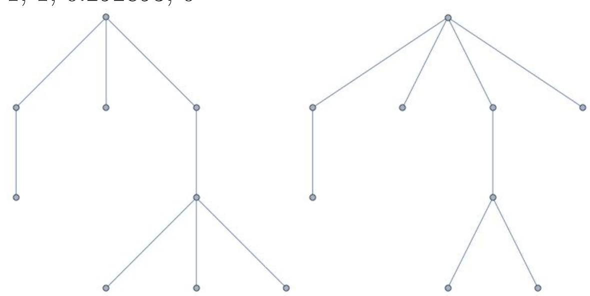


4 Zero Conductivities

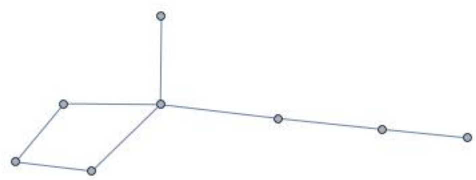
Eigenvalue	*BV*	*IV*	*R*1	*G*1	*R*2	*G*2	*Total*	*Radius*	*Diameter*	*Mean Graph Distance*
1	4	2	3(1)	4(3)(1)	0	2	16	3	5	2.611, 2.556
2	3	2	2(2)	4(2)(2)	1	3	8	3	5	65/28
3	4	2	2(2)	4(2)(2)	0	2	8	3	6	2.321
4	3	2	2(2)	4(2)(2)	1	2	8	3	4	2.107
5	4	2	2(2)	4(2)(2)	0	1	8	3	6	2.889
6	3/4	2	3(1)	4(3)(1)	1/0	2	2	3, 2	5, 3	2.321, 1.733
7	4	3	2(1)(1)	3(3)(2)	0	2	24	3	6	2.833
8	3	3	2(1)(1)	3(2/3)(2)	1	2	24	3	4	2.179
9	4	3	2(1)(1)	3(3)(2)	0	2	8	3	6	3
10	3/4	2	3(1)	4(3)(1)	1/0	3/2	14	3, 2	5, 4	2.25, 2, 2.321
11	3	3	2(1)(1)	3(3)(2)	1	3/2	27	3	5	2.536, 2.464
12	2	3	2(1)(1)	3(3)(2)	2	3	4	3	3	1.810
13	4	3	2(1)(1)	3(3)(2)	0	2	4	2	4	1.867
14	4	2/3	2(2)/2(1)(1)	(4/3)(2/3)(2)	1	3	8	*	*	*
15	2	2	2(2)	4(2)(2)	2	4	2	2	4	1.905

Eigenvalues:

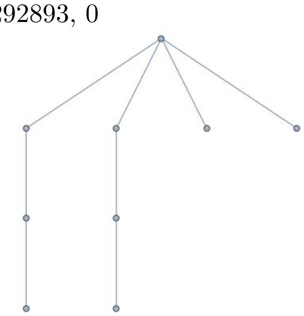
1. 1.70711, 1, 1, 1, 0.292893, 0



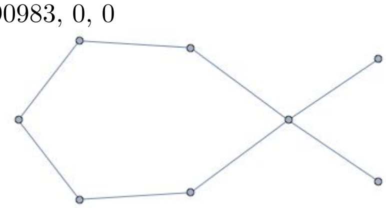
2. 2, 1.61237, 1, 0.387628, 0, 0



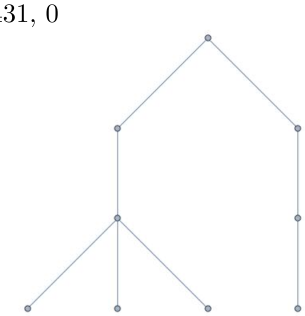
3. 1.70711, 1.5, 1, 0.5, 0.292893, 0



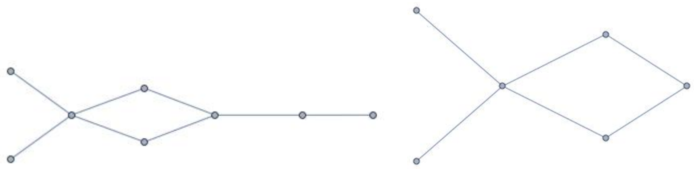
4. 1.80902, 1.5, 1, 0.690983, 0, 0



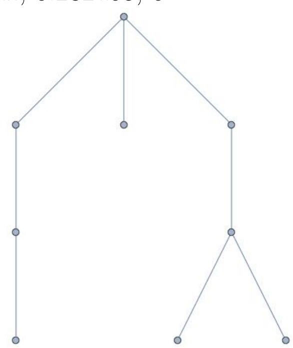
5. 1.79057, 1, 1, 1, 0.209431, 0



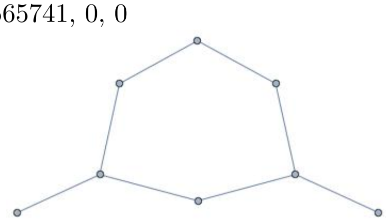
6. 2, 1.40825, 1, 0.591752, 0, 0



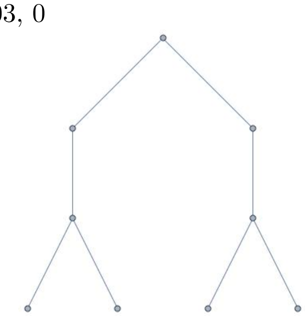
7. 1.76759, 1.4342, 0.565741, 0.232408, 0



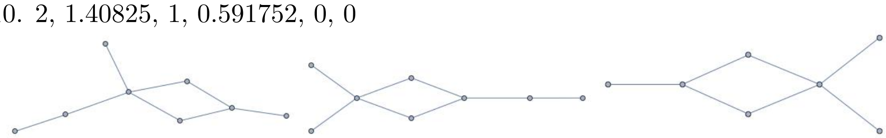
8. 1.76759, 1.67, 1, 0.565741, 0, 0



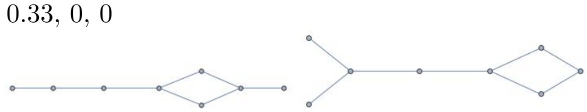
9. 1.8165, 1, 1, 1, 0.183503, 0



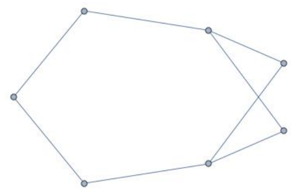
10. 2, 1.40825, 1, 0.591752, 0, 0



11. 2, 1.67, 1, 0.33, 0, 0

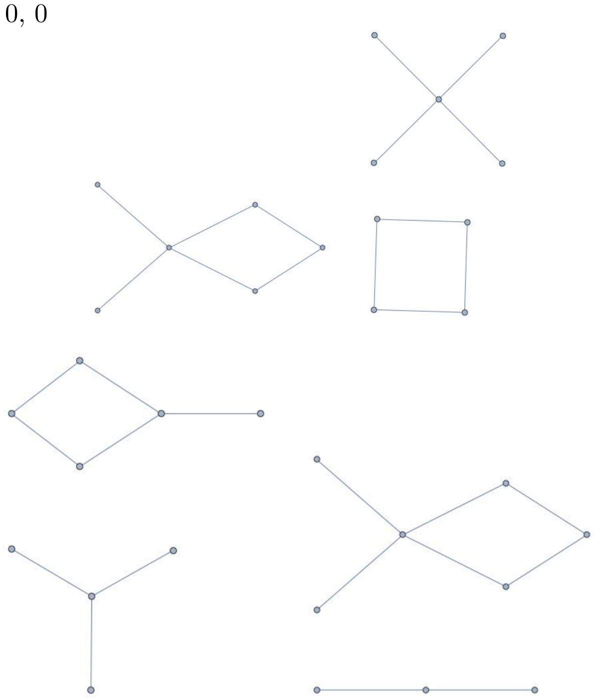


12. 2, 1.67, 1.33, 0, 0, 0

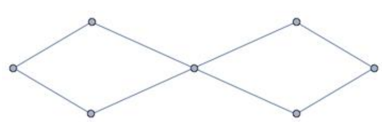


13. 2, 1.33, 1, 0.67, 0, 0

14. 2, 1, 1, 1, 0, 0



15. 2, 2, 1, 0, 0, 0



It was unclear from simply looking at the number of boundary vertices and the degree of the vertices what the differences are between the different grouping. The mean graph distance seems to offer somewhat of an explanation for the differences between groups for networks with 4 zero conductivities, but more needs to be done to understand the differences between groups. My plan going forward is to try analyze the graphs was bipartite and compare the characteristics between groups.

5 Future Research

1. Can a process similar to the one used in Section 3 be used to find the malfunctioning resistor in an open or short circuit where the nonzero/noninfinite conductivities are not all equal?

2. Is there an alternate explanation for why we observe the same eigenvalues no matter the placement of the zero/infinite resistance edge that is based on the Kirchhoff matrix or the response matrix?

3. Can the number of eigenvector groups and division of networks into these groups be predicted?

4. What is the physical meaning of the magnitude of the eigenvalue?

5. Are the results generalizable to other networks with symmetries?

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