Solutions to Math 120 A Winter 2025 Final Exam

1. (a) $Z(0,0)$, $V(0,400)$, $B(300,0)$, $C(900,-800)$, $M(-2100,-800)$

   (b) When $t=0$, $x=900$ and $y=-800$ so

$$x = 900 + at,\ y = -800 + bt$$

   When $t=80$, $x=300$ and $y=0$ so

$$x = 900 - \frac{15}{2}t,\ y = -800 + 10t$$

   (c) Suleiman in Vienna when

$$0 = 900 - \frac{15}{2}t,\ 400 = -800 + 10t$$

   so $t = 120$ days. Charles has $120 - 14 - 10 = 96$ days to get there. The distance from Madrid to Vienna is

$$\sqrt{(400+800)^2 + (0+2100)^2} = 100\sqrt{585}$$

   so his speed must be $\frac{100\sqrt{585}}{96} \approx 26.01$ miles per day.

2. The radius is $61/2 = 30.5$ meters. The linear speed is 2.7 km/hr = 2700 m/hr = 2700/60 = 45 m/min. The angular speed is $45/30.5 = 90/61$ radians per minute. With the x-axis on the ground and the y axis going through the center of the wheel

$$x = 30.5\cos\left(\frac{90}{61}t - \frac{\pi}{2}\right),\ \ y = 34.5 + 30.5\sin\left(\frac{90}{61}t - \frac{\pi}{2}\right)$$

   With the origin at the center of the wheel

$$x = 30.5\cos\left(\frac{90}{61}t - \frac{\pi}{2}\right),\ \ y = 30.5\sin\left(\frac{90}{61}t - \frac{\pi}{2}\right)$$

   (we don't need x really) When you are 45 meters from ground we solve

$$y = 34.5 + 30.5\sin\left(\frac{90}{61}t - \frac{\pi}{2}\right) = 45 \text{ or } y = 30.5\sin\left(\frac{90}{61}t - \frac{\pi}{2}\right) = 45 - 34.5$$

   which are the same, of course. So,

$$\sin\left(\frac{90}{61}t - \frac{\pi}{2}\right) = \frac{10.5}{30.5}$$

   which gives us the solutions

$$\frac{90}{61}t - \frac{\pi}{2} = \arcsin\left(\frac{10.5}{30.5}\right), \pi - \arcsin\left(\frac{10.5}{30.5}\right), 2\pi + \arcsin\left(\frac{10.5}{30.5}\right), 2\pi + \pi - \arcsin\left(\frac{10.5}{30.5}\right), \ldots$$

   adding $\pi/2$ to both sides and switching to decimals

$$\frac{90}{61}t = 1.922, 4.3609, 8.2054, 10.6441, 14.4886$$

   which gives, after rounding to decimal places,

$$t = 1.30, 2.91, 5.56, 7.21, 9.92.$$

3. Line through $P(0,-1)$ and $R(3,8)$ has equation

$$y = 3x - 1.$$

Line through $Q(2,1)$ and perpendicular to the previous line has equation

$$y = -\frac{1}{3}x + \frac{5}{3}$$

The two intersect at T where

$$3x - 1 = -\frac{1}{3}x + \frac{5}{3}$$

so $x = 0.8$ and $T(0.8, 1.4)$.
The parabola has to go through the points $P(0,-1)$, $R(3,8)$, and $Q(2,1)$ so it has equation

$$y = ax^2 + bx - 1$$

where

$$8 = 9a + 3b - 1 \qquad \text{and} \qquad 1 = 4a + 2b - 1$$

so $a = 2$ and $b = -3$ so the parabola equation is

$$y = 2x^2 - 3x - 1$$

The point T is where the parabola and the second line intersect

$$2x^2 - 3x - 1 = -\frac{1}{3}x + \frac{5}{3}$$

which gives $x = 2$ (for point Q) and $x = -2/3$ for point

$$S\left(-\frac{2}{3}, \frac{17}{9}\right).$$

4.

$$y = 5\sin\left(\frac{2\pi}{3}(x-1)\right) + 4, \quad \left(\frac{x-1}{3}\right)^2 + \left(\frac{y+3}{2}\right)^2 = 1, \quad y = 4\left(\frac{3}{2}\right)^x, \quad y = 3|x-4| - 3$$

5. (a) The tank has capacity $V = \frac{1}{3} \cdot \pi \cdot 5^2 \cdot 8 = \frac{200}{3}\pi$ cubic meters. To find the volume of the oil, we need the radius at the surface which can be found from

$$\frac{5}{8} = \frac{r}{4}$$

(both are tangents of the angles for the right triangle). So $r = 2.5$ and the oil has volume

$$V_{\text{oil}} = \frac{1}{3} \cdot \pi \cdot 2.5^2 \cdot 4 = \frac{25}{3}\pi.$$

Therefore,

$$\frac{\frac{25}{3}\pi}{\frac{200}{3}\pi} = 0.125 = 12.5\%$$

is full.

(b) The oil will have the same volume, but now it has the shape of another cone with radius r and height h, which, from similar triangles again, satisfy

$$\frac{3}{10} = \frac{r}{h}$$

so $r = \frac{3h}{10}$. So, the volume of the oil is

$$\frac{25}{3}\pi = \frac{\pi}{3}\left(\frac{3h}{10}\right)^2 \cdot h$$

which gives $h = \sqrt[3]{\frac{2500}{9}}$.

6. Solve for x.

(a) Isolate the ln:

$$\ln(2+3x) = 3$$

exponentiate

$$2+3x = e^3$$

to solve $x = \frac{e^3-2}{3}$.

(b) Using laws of exponents and rearranging

$$6\left(e^x\right)^2 + 13e^x - 5 = 0$$

so using the Quadratic Formula

$$e^x = \frac{-13 \pm \sqrt{169 - 4\cdot 6\cdot(-5)}}{12} = \frac{-13 \pm 17}{12} = \frac{1}{3}, -\frac{5}{3}.$$

Since $e^x > 0$ we must have $e^x = \frac{1}{3}$ so $x = -\ln 3$.

(c) $2 - x + |3x-5| = 1$ Here we consider the two cases. First, if $3x - 5 \geq 0$ then we solve

$$2 - x + 3x - 5 = 1$$

which gives $x = 2$ (which satisfies $3x - 5 \geq 0$.) The second case is when $3x - 5 < 0$ then we solve

$$2 - x - 3x + 5 = 1$$

which gives $x = 3/2$ (which satisfies $3x - 5 < 0$.)